

THE
RELATIONS OF THE GLOBULAR CLUSTERS
AND SPIRAL NEBULÆ TO THE
STELLAR SYSTEM

AN ATTEMPT TO ESTIMATE THEIR PARALLAXES

INAUGURAL DISSERTATION

BY

KNUT LUNDMARK

WITH 1 PLATE AND 11 FIGURES IN THE TEXT

STOCKHOLM

ALMQVIST & WIKSELLS BOKTRYCKERI-A.-B.

1920

Diss. Uppsala
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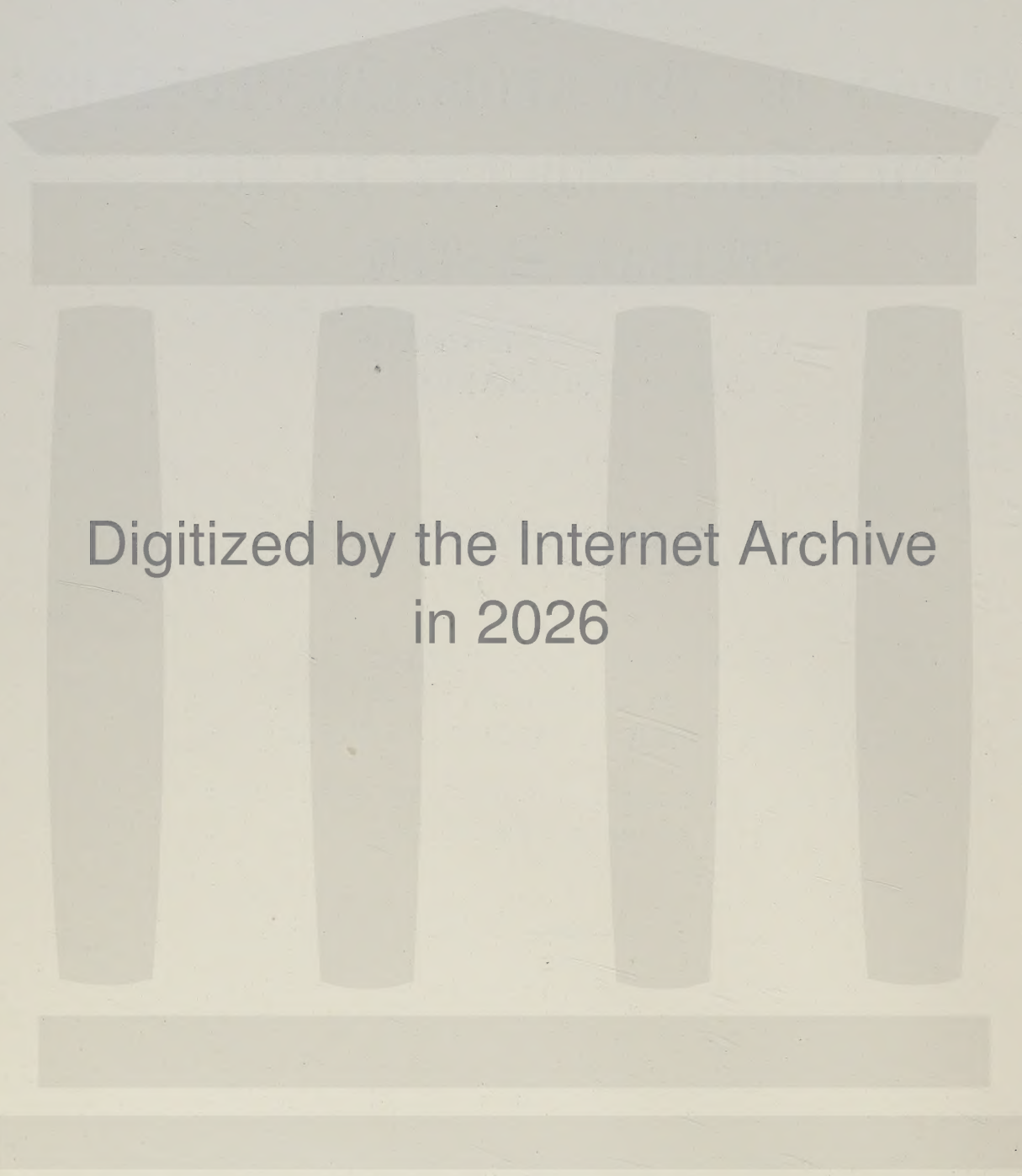
BY PERMISSION OF

THE PHILOSOPHICAL FACULTY OF UPSALA

TO BE PUBLICLY DISCUSSED IN THE LECTURE HALL OF THE PHYSICAL INSTITUTE

FEBR. 21TH 1920, AT 10 O'CLOCK A. M.

STOCKHOLM
ALMQVIST & WIKSELLS BOKTRYCKERI-A.-B.
1920



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PREFACE.

The beautiful contributions to our knowledge of the structure and extent of the stellar system given through the researches of SEELIGER, KAPTEYN, SCHWARZSCHILD and others, have conferred a higher actuality to the difficult problems regarding the distribution in space of the clusters and spiral nebulae and their relations to our galactic system. Concerning these interesting objects our knowledge has during the last years been carried considerably forward through a series of important observational works performed principally at the great American observatories.

The question regarding the distances of these objects is the dominating problem within this branch of Astronomy, and on account of their unmistakeable relation to the Galaxy, the solution of this problem is important not only as giving a wider knowledge of the objects themselves, but also regarding our general conception of the stellar Universe.

Already some years ago I have collected material for a resume of the different methods practicable for estimating the parallaxes of the said objects. The publication of the researches of SHAPLEY and CHARLIER regarding the globular clusters have given the external impulse to the elaboration of the results presented below.

The following paper was worked out at the Upsala Observatory in the spring of 1919 and presented to the Royal Swedish Academy of Sciences for publication in its Proceedings. Owing to the typographers' strike in the summer of 1919, the paper could, however, not be put in press before December in the same year.

The investigation in the first place aims at a discussion, with aid of the existing observation material, of the different ways by which it may be possible to attain an estimate of the parallaxes of the globular clusters and spiral nebulae. The present moment may appear somewhat premature for undertaking such an attempt, but, nevertheless, the general state of things in this domain of Astronomy seems to indicate that the results of to-day are definitive in such manner as not giving much hope of considerable further advances in this direction within a fairly long time.

In the second place, the present investigation also intends, by a discussion of the results already obtained, to call attention to such questions in the astronomy of clusters and nebulae where it might be possible, by reiterated efforts and systematic employment of our instrumental resources, to obtain new results throwing light upon the problem of distances.

It is to me a pleasant duty here to express my hearty thankfulness to my teacher and friend, Professor ÖSTEN BERGSTRAND for his valuable and stimulating instruction in various branches of Astronomy and for the interest he has always shown in my studies during the time I have been working at the Observatory. To my friend Professor HUGO VON ZEIPPEL I am in very great obligation for his interested perusal of this paper and for the kind interest with which he has followed my researches. My colleague Mr. B. LINDBLAD I beg to offer my thanks for his interest in my investigations and for our agreeable collaboration in the spectral study of the clusters and nebulae. Further it is to me a precious duty to tender my heartfelt thanks to Mr. STEN ASKLÖF for his valuable assistance at the numerical calculations and for the keen interest he has in other respects shown for the present investigation. Likewise I am under a great obligation to Mr. KARL O. LITTMARCK for the interesting Apex value he has calculated and allowed me here to communicate.

To Mr. PER COLLINDER, of the Hydrographic Office, I am greatly indebted for his patient assistance in reading the English proofs and for his translation of parts of the manuscript. Finally, I beg to express my thanks to all my other colleagues and friends that have in one way or other furthered my investigations.

It may here also be permitted to mention that my researches have been considerably furthered by the purchase of the 6-inch twin telescope to the Upsala Observatory, which was made possible by the generous gift of an anonymous donator.



KUNGL. SVENSKA VETENSKAPSAKADEMIENS HANDLINGAR. Band 60. N:o 8.

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COMMUNICATED MAY 28TH 1919 BY K. BOHLIN AND H. VON ZEIPPEL

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I. The Parallaxes of Globular Clusters.

Most celestial objects, which, as regards their distribution in the sky, prove to be dependent on the galactic plane, seem to be uniformly distributed in galactic longitudes. This is the case with the eclipsing variables, *c* and *ac* stars, stars of Type IV and δ -Cephei variables⁸⁴. The Helium-stars and the stars belonging to class $Oa_0 - Oe_0$, on the other hand, are concentrated towards one side of the ring of the Milky Way^{37, 84} (λ 248° and 305° resp.) and, again, for the planetary nebulae we also find a concentration corresponding to the $Oa - Oc$ stars. A still more strongly pronounced dependence on the galactic longitude is shown by the globular clusters and the spiral nebulae. The distribution in the sky of these objects has recently been studied by, among others, BAILEY^{5, 9}, BOHLIN²⁴, CHARLIER^{37, 39}, FATH⁶⁶, HARDCASTLE⁷⁴, HINKS^{89, 90}, HERTZSPRUNG⁸⁴, MELOTTE¹²⁸ and PERRINE¹⁴⁶. Concerning globular clusters it has come forth as the main result of those researches, that they occur exclusively in a hemisphere, whose pole has the galactic coordinates $\lambda + 296^\circ$, $\beta - 9^\circ$, and that they are particularly strongly concentrated in the great star cloud in Sagittarius³⁹.

The spiral nebulae, on the other hand, were formerly assumed to be symmetrically distributed round the galactic plane and concentrated at its poles, but otherwise uniformly distributed in galactic longitude. While the globular clusters seem to form a system, the members of which may be almost all known¹¹, and whose number, it seems, is not above 80, the spiral nebulae are certainly to be estimated to millions⁵³, and to formulate a general law, rendering their distribution in the sky, will probably not be possible out of the scanty material, that, as yet, is at our disposal for researches into their distribution. Nevertheless, it is evident from HARDCASTLE'S-HINK'S⁷⁴ and from FATH'S⁶⁶ researches, that in the law expressing their distribution galactic longitude must also form part as an important parameter. MAX WOLF'S studies with the aid of stellar photography of small nebulae also show, that the spiral nebulae in several regions gather in cluster-formations (at the pole of the Milky Way²³³, in Perseus²⁴⁵, in Cetus). We can certainly expect to find a very complicated structure in the doubtlessly gigantic universal system, which is formed by the spiral nebulae.

The enigmatic distribution of globular clusters and spiral nebulae has since the days of HERSCHEL tempted many scientists to occupy themselves with more or less hypothetic deliberations as to the distribution in space of these objects. Already

Sir W. HERSCHEL⁷⁷⁻⁸³ attempted with the aid of the magnification, which in his great reflector was required in order to distinguish the stars in the particular globular clusters, and the thereby calculated »space-penetrative power» of the instrument, to determine the distribution in space of these objects. Afterwards many results have been obtained, which throw some light upon the question of the real position in space of the globular clusters and spiral nebulæ, but no decision can yet be said to have been made as to the latter. On the other hand, very remarkable results regarding the distribution in space of the globular clusters are given through the great researches made by SHAPLEY¹⁸⁸⁻²⁰⁷ and CHARLIER³⁷⁻³⁹. Although their results strike us as very contradictory, there seems to exist some possibility of explaining — at any rate partly — the origin of the comparatively small distances for globular clusters, calculated by CHARLIER.

In a series of extensive investigations SHAPLEY has determined individual parallaxes for 69 globular clusters, or all objects that have with certainty been identified as such. Almost at the same time, CHARLIER in an extensive treatise has solved this problem by another method, and he, thereby, for his clusters gets distances, which fall below the corresponding values found by SHAPLEY by 40—290 times. With regard to the great importance of this question I have therefore considered it justifiable here to publish a few small calculations and deliberations, which may possibly contribute to the knowledge of the relations of globular clusters and spiral nebulæ to the stellar system.

The starting-point for SHAPLEY's parallax-measures is the remarkable relation between the length of period and the absolute magnitude of the δ -Cephei variables, which was first discovered by Miss LEAVITT¹¹⁷. After having with great accurateness established this connection anew from more recent data, SHAPLEY, presuming that the considerable number of cluster-variables (blink-variables, antalgol stars) found in certain globular clusters are to be considered as δ -Cepheids with the same qualities as the ones found in the Milky Way, deduces the absolute magnitude M for these, from which the parallax π comes forth, according to the well-known formula:

$$M = m + 5 + 5 \log \pi. \quad (1)$$

After having by this method obtained parallaxes for the globular clusters in which cluster-variables have been observed, or N. G. C. 5272, 5904, 6205, 6656, 7078 and 5139, and the small Magellanic cloud, SHAPLEY finds that the mean of the absolute magnitude of the 25 brightest stars in a globular clusters has a constant value of

$$M = -1,51 \pm 0,3.$$

By means of this law empirically found, SHAPLEY obtains the parallaxes of the 28 globular clusters, for which it has been possible to determine the apparent magnitude of the brightest stars.

A necessary condition for obtaining good values for π by means of this method is, that m is fixed in an absolute photometrical scale. The extensive work that has been performed at the Harvard and Mount-Wilson Observatories for the fixing of a photometric scale round the pole, has a great importance for researches of this kind. For the star magnitudes here in question these sequences can certainly be regarded as representing a scale, that may very nearly be considered as the exact one.

Finally, by using the 28 globular clusters for which the parallax has been thus obtained from variable stars and from magnitudes for the brightest stars, SHAPLEY determines the connection between the apparent diameter and the parallax; and he then obtains, by means of this connection, the parallaxes for the remaining 41 globular clusters through measures of the apparent diameter, performed by him and Miss HELEN DAVIS.

Treating the question of the distances of the globular clusters, CHARLIER starts from the presumption that the absolute dimensions of these objects vary within rather narrow limits, and that the apparent diameter must consequently be a proper measure of the distance. This supposition is justified by SHAPLEY's statement in his sixth paper. By using BAILEY's values for the diameters of globular clusters, CHARLIER thus obtains their relative parallaxes, and now there only remains the determining of the scale-value for obtaining the absolute dimensions of the distances. Determining this constant, CHARLIER supposes, on account of the concentration of the globular clusters in the star cloud in Sagittarius and of their strongly symmetrical distribution in relation to the galactic plane, that they are to be regarded as related to the Galaxy. This supposition he regards as having a strong support in the fact that the apparent distribution of the planetary nebulae is accordant with that of the globular clusters. Regarding the correctness of this statement one may perhaps be in doubt, on account of the fact that in the star cloud in Sagittarius the concentration of planetary nebulae is not so pronounced as that of globular clusters, and, besides, in that hemisphere of the sky, where only 15 % of all the globular clusters are to be found, there are no less than 40 % of all the planetary nebulae. Furthermore, WOLF, in his Catalogues of Nebulae, classifies rather a great number of nebulae as planetary ones (in WOLF's Nebelliste No. 3 thus occur 30, in No. 5 24 planetary ones). On the contrary, he seems not to have found one single globular cluster in the same regions, a fact which shows that the planetary nebulae concentrate also in regions where globular clusters occur only separate or not at all, and that, accordingly, no great concordance, except the mutual concentration in the star cloud in Sagittarius, can with certainty be said to exist between the apparent distribution of globular clusters and planetary nebulae. Yet, there seems to exist some strong reasons for assuming that globular clusters are related to the Galaxy, and, consequently, with the acceptance of CHARLIER's suppositions, the question of the determination of their distances can be reduced to the question of the determination of the dimensions of the galactic system, by which, however, a rather great amount of uncertainty is brought in, as the question of the dimensions of the Milky Way can scarcely be regarded as satisfactorily solved as yet.

CHARLIER supposes, that the 800 bright Helium-stars, for which he has, by the method employed by him, determined individual parallaxes³⁸, form a skeleton to the Galaxy. Partly by supposing that the centre of the system of globular clusters is situated as far from the galactic plane as the centre of the Helium-star system, and by supposing the dispersion $\sigma(z)$ in the same direction to be as great in both systems, CHARLIER obtains two determinations of the scale-value.

In order to get a synoptical comparison between CHARLIER's and SHAPLEY's investigations, I have put together their results in the adjoining table, where CHARLIER's distances are reduced to parallaxes and the ratio between CHARLIER's and SHAPLEY's parallaxes is given in the fourth column. As CHARLIER has employed only BAILEY's diameter-values for obtaining the relative parallaxes, I have in Col. 5—10 put together determinations based upon the photographic plates that were at

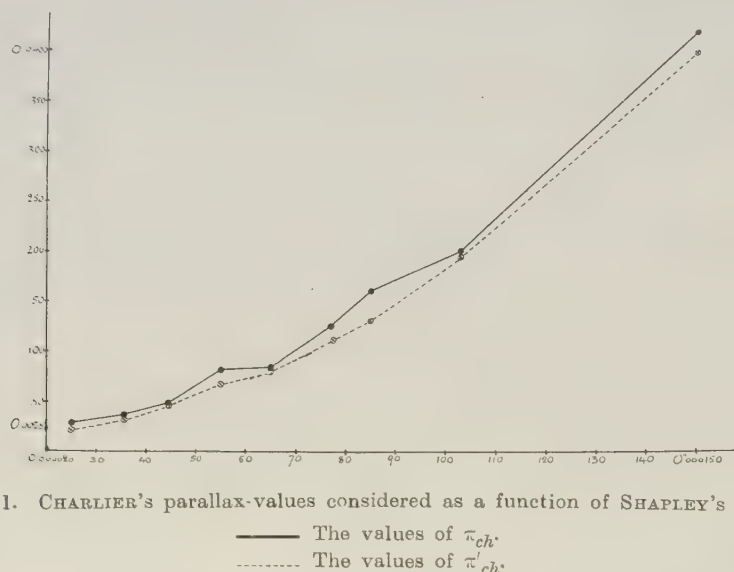


Fig. 1. CHARLIER's parallax-values considered as a function of SHAPLEY's values.

— The values of π_{ch} .
 - - - The values of π'_{ch} .

my disposal. In this table, *B*, *M*, *P*, *Sh*, *D* and *Lk* denote the estimations of the diameter made by BAILEY, MELOTTE, PERRINE, SHAPLEY, DAVIS and by the present writer using the WOLF-PALISA charts, the Harvard Annals, Vol. 26 and 38, and, in some cases, my own plates. Col. 12 contains the relative parallaxes calculated in accordance with CHARLIER's formulæ out of the mean given in Col. 11 and multiplied with a constant ϵ' , with the determination of which I am going to occupy myself in the following. Col. 13 contains the ratio between CHARLIER's parallaxes, thus reduced, and SHAPLEY's values.

From the table it is evident that the ratio between π_{ch} and π_{sh} (Col. 4) is dependent on the value of the parallax. If the parallaxes are arranged according to their numerical value, and if means are calculated, the above curve is obtained, which gives the relation between the parallaxes in the two systems.

We are now going to try to determine ϵ' without employing any of the suppositions, made by CHARLIER and SHAPLEY, in order to obtain, if possible, a decision as to which of the two parallax-systems is nearer to truth. In the Popular

Table I.

N. G. C.	π_{ch}	π_{sh}	$\frac{\pi_{ch}}{\pi_{sh}}$	D i a m e t e r							π'_{ch}	$\frac{\pi'_{ch}}{\pi_{sh}}$
				B	M	P	Sh	D	Lk	Mean		
104	0'',0387	0'',000148	261	30'	42'	—	27',6	26',0	20'	29',0	0'',0413 ε'	253 ε'
288	129	53	243	10	12	—	4,4	4,0	—	7,6	98	185
362	129	66	195	10	10	—	6,6	5,4	6	7,6	98	148
1261	39	39	100	3	5	—	2,5	2,6	—	3,3	43	110
1851	52	58	90	4	8	—	5,0	4,8	—	5,5	71	122
1904	52	39	133	4	4,5	—	2,2	2,4	2	3,1	40	103
2298	39	41	95	3	2	—	2,7	3,0	—	2,7	35	85
2808	77	59	131	6	5	—	4,9	5,2	—	5,3	68	115
3201	103	68	152	8	15	—	6,3	6,6	4	8,0	103	152
4147	—	19	—	1	1	1,5	0,8	0,8	1,5	1,1	14	100
4372	193	88	219	15	12	—	9,3	10,8	—	11,8	152	173
4590	64	62	103	5	8	—	5,8	4,8	—	5,9	76	123
4833	77	61	126	6	6	—	4,9	5,6	—	5,6	72	118
5024	64	53	121	5	5	5	5,4	5,6	5	5,2	67	126
5139	451	153	295	35	45	15	28,0	32,0	27	31,0	400	261
5272	155	72	215	12	18	7	8,3	5,8	7	9,7	125	174
5286	129	51	253	10	6	—	4,5	3,6	—	6,0	77	151
5634	—	33	—	1	1,5	—	2,0	2,2	3,0	1,7	22	67
5897	103	67	154	8	6	—	6,2	6,4	—	6,7	86	128
5904	155	80	194	12	15	10	8,2	9,1	8	10,4	134	167
5986	52	48	108	4	5	—	4,1	3,3	—	4,1	53	110
6093	64	50	128	5	5	3	8,2	9,1	3	5,5	71	134
6101	39	47	83	3	5	—	4,1	3,3	—	3,8	49	104
6121	232	88	264	18	20	—	11,0	10,0	8	13,4	173	197
6144	—	41	—	—	5	—	2,8	2,8	—	3,9	50	122
6171	45	62	73	3,5	7	5	5,0	5,8	—	5,3	68	110
6205	193	90	214	15	12	10	11,0	10,2	—	11,6	150	167
6218	129	81	159	10	9	7	8,8	8,8	7	8,7	112	138
6229	—	23	—	0,4	—	2	1,2	1,0	0,5	1,1	14	61
6235	—	20	—	—	1,5	—	1,2	1,0	—	1,2	15	75
6254	155	83	187	12	10	7	5,9	6,4	8	7,9	102	123
6266	64	66	97	5	8	4	5,9	6,4	3,5	5,4	70	106
6273	52	63	83	4	5	—	5,4	5,6	—	5,0	64	102
6284	26	27	96	2	1,5	—	1,9	1,4	—	1,7	22	81
6287	32	23	139	2,5	1,5	1,5	1,4	1,3	—	1,6	21	91
6293	39	38	103	3	2,5	—	2,2	2,9	—	2,7	35	92
6304	26	31	84	2	1,5	—	1,9	2,0	—	1,9	24	77
6316	—	19	—	1	—	—	1,0	1,0	—	1,0	13	68
6333	64	40	160	5	3	3	3,5	3,0	3,1	3,4	44	110
6341	64	81	79	5	8	7	7,6	8,4	9	7,5	97	120
6356	26	26	100	2	1,5	2	1,8	2,0	1,5	1,8	23	88
6362	64	77	83	5	10	—	8,4	7,3	—	7,7	99	129
6388	39	36	108	3	3	—	2,2	2,5	1,5	2,4	31	86
6397	258	120	215	20	18	—	16,2	18,0	(13)	18,0	232	193

N. G. C.	τ_{ch}	τ_{sh}	$\frac{\tau_{ch}}{\tau_{sh}}$	D i a m e t e r							π'_{ch}	$\frac{\pi'_{ch}}{\tau_{sh}}$
				B	M	P	Sh	D	Lk	Mean		
6402	0,0052	0,000043	121	4	—	5	3,8	3,1	5	4,2	0,0054	126 ϵ'
6441	26	22	118	2	1,5	—	1,2	1,3	—	1,5	19	86
6541	103	68	152	8	7	—	6,2	6,8	—	7,0	90	132
6584	32	38	84	2,5	2	—	2,6	2,6	—	2,4	31	82
6624	32	35	91	2,5	2	2	2,2	2,4	—	2,2	28	80
6626	52	54	96	4	5	4	4,2	5,2	2,1	4,0	52	96
6637	39	47	83	3	2	3	3,6	3,3	—	3,0	39	83
6638	26	29	90	2	—	—	1,8	1,6	0,7	1,5	19	66
6642	—	26	—	1	1	8	1,2	1,2	0,7	2,2	28	108
6652	26	32	81	2	1	—	2,0	2,0	—	1,8	23	72
6656	155	118	131	12	20	10	17,5	14,5	12	14,3	185	157
6681	52	55	95	4	2,5	2,5	4,6	4,4	—	3,6	46	84
6712	39	32	122	3	2,5	3	2,0	2,2	3,3	2,7	35	109
6715	26	62	42	2	3	2	5,2	5,7	—	3,6	46	74
6723	103	79	130	8	8	—	9,2	7,3	6	7,3	94	119
6752	193	114	169	15	18	—	14,4	16,8	—	16,1	208	182
6779	32	40	80	2,5	1,5	3	2,6	2,2	1,5	2,5	32	89
6809	193	100	193	15	14	10	13,2	11,7	11	12,5	161	161
6864	32	22	145	2,5	2	2	1,5	1,6	—	1,9	24	109
6934	—	30	—	1,5	4	2	1,5	1,4	—	2,1	27	90
6981	32	34	94	2,5	2	2,5	2,4	2,2	—	2,3	30	88
7006	—	15	—	—	—	—	0,7	0,8	—	0,8	10	67
7078	129	68	190	10	6	7	5,2	4,8	6	6,5	84	124
7089	103	64	161	8	8	8	6,8	7,3	8,2	7,7	99	155
7099	0,0103	0,000058	178	8	6	6	4,7	4,6	6,8	6,0	0,0077 ϵ'	133 ϵ'
Clusters not considered as globulars by SHAPLEY												
2099	0,0322	—	—	25	20,0	—	—	—	24,0	23,0	0,0296 ϵ'	—
5927	129	—	—	10	4,5	—	—	—	—	7,2	93	—
6522	—	—	—	1,5	—	—	—	—	—	1,5	19	—
6528	—	—	—	0,7	—	—	—	—	—	0,7	9	—
6553	26	—	—	2	1,5	—	—	—	—	1,7	22	—
6558	—	—	—	1	—	1	—	—	—	1	13	—
6569	26	—	—	2	1,5	1,5	—	—	12,0	1,7	22	—
6705	0,0155	—	—	12	10,0	—	—	—	13,1	12,0	155	—
7214	—	—	—	0,3	—	—	—	—	—	0,3	0,0004 ϵ'	—

Astronomy, for Jan. 1918, SLIPHER has given the determinations of the total radial velocity of 10 globular clusters²¹⁵, which will be found in Table III. These velocities are found to be of another order of magnitude than the radial velocities of the stars, a circumstance, which alone goes to prove that globular clusters cannot possibly be in our immediate vicinity. Under the supposition that no preference of motions exists among the globular clusters, a comparison between the cross motion τ_m and the mean radial velocity V_m gives the mean parallax of the globular clusters π_m according to the following formula³²:

$$\pi_m = 4,74 \frac{\tau_m}{V_m}. \quad (2)$$

The sources at my disposal for a determination of the total proper motion of globular clusters are, unfortunately, very insufficient, partly because it has not been possible to perform any reliable measurements of positions of stars in these objects, before the photographic measuring methods had been worked out and acknowledged, partly because astronomers have as a rule been more interested in the relative motions in the globular clusters, and less devoted themselves to the determination of the translational motion and even not always taken care that the measurements may be connected with such surrounding star that can be regarded as not being physical members of the group.

For the following globular clusters there exist measures of the positions of the individual stars in the cluster.

M 3. N. G. C. 5272. One of the most extensive measurements yet published of the positions in a globular cluster is the one performed by VON ZEIPPEL for M 3²⁵¹. Unfortunately there exist no earlier measurements, and the epochal difference is probably too small to justify a new measuring, but when the work is later on to be repeated, an important contribution to our knowledge of the translation and the internal motions in this stellar system can be expected.

M 13. N. G. C. 6205. Measured by SCHEINER¹⁷⁶, LUDENDORFF¹¹⁹, H. C.¹⁵⁷ and W. E. PLUMMER¹⁵⁸. The measurements of the last author have not yet been completely published. A few small differences of systematic character seem to remain between the different measurements. Thus LUDENDORFF finds a systematic difference between SCHEINER's declinations and his own of the form $d\delta = -0'',06 \Delta\delta$, which he is inclined to ascribe not to motion but only to a difference in scale-value. LUDENDORFF's and W. E. PLUMMER's discussion of the 4 series of measurements of M 13 show, that also this globular cluster must be considered as having a very slight lateral motion.

M 37. N. G. C. 2099. Three series of measurements performed by NORDLUND¹³⁴, GIEBELER⁶⁹ and JOY⁹⁹ are in hand for this globular cluster. The comparison made by GIEBELER between his own measurements and those performed by NORDLUND show a rather great difference in declination, evidently owing to a magnitude equation originating from a prolongation of NORDLUND's photographic images. Partly on this account, and partly as the epochal differences are comparatively small, these measurements cannot be used for an estimate of the total proper motion of the cluster, which motion, however, judging from GIEBELER's discussion, is evidently very small. Thus JOY finds by his measurements, from a comparison between plates taken with an interval of 11,05 years, that the total proper motion of the cluster, relatively to the reference stars, has the value:

$$\mu = 0'',0066; \quad p = 46^\circ.2.$$

Any considerable importance will scarcely be attached to this determination, but it will nevertheless be permitted, in JOY's value for μ to see an upper limit for the total proper motion of the cluster.

M 92. N. G. C. 6341. At a comparison between SCHULTZ's¹⁸² micrometrical measurements and his own photographic ones, BOHLIN²² thought to have found remarkable proper motions in this group. Micrometric and photographic measures performed by BARNARD¹³⁻¹⁵ have, however, shown, that the differences found by BOHLIN cannot be due to proper motion, but are probably to ascribe to SCHULTZ's incomplete instrumental resources. BARNARD finds only two stars, for which the proper motion amounts to a considerable value:

μ	p	
0",083	225°,3	} (photographic measures)
085	222,0	
0,065	181,4	visual »

The total displacement of the cluster certainly amounts to a far lower value than the proper motions of these stars, which perhaps do not belong to the cluster, but probably are only projected against it.

ω Centauri. N. G. C. 104. In recent years the stereocomparator has got an important use in the searching for and measuring of the proper motion of the stars. INNES⁹⁴ has shown that particularly good results can be obtained if the stereocomparator with the blink-arrangement is used. He has together with VOÛTE measured proper motions in ω Centauri and finds, out of the 2000 stars on their plates belonging to the cluster, only 30 having a proper motion of 1",0—1",5 during 23 years. $\mu = 0'',05$ must consequently be the upper limit for the individual motions within the group. INNES has never stated that these stars are members of the group, and it is probable that they do not belong to the cluster, but are perhaps only situated in front of it. The individual motions within the system will consequently be diminutive. Now the possibility remains that the cluster has a motion in common with the background of the surrounding faint stars; against this supposition, however, speaks the fact that the proper motions found by INNES agree well with the proper motions, which for part of the stars in question have been determined out of the Cape meridian observations. As these latter proper motions are to be considered, in the case in question, as absolute, it is evidently probable that the proper motions found by INNES are also absolute, and that, consequently, the background and the cluster cannot have any common systematic motion. Even an amount of 0'',01 is probably altogether too high as a hypothetical upper-limit value for the total proper motion.

M 11. N. G. C. 6705. Photographically measured in 1896 by STRATONOFF²¹⁹, micrometrically by LAMONT¹¹⁰ in 1838—39, and by HELMERT⁷⁶ in 1869—70. The slight systematic differences that remain between STRATONOFF's and LAMONT's and HELMERT's measurements, and of which a total displacement of the group might possibly form part (page 132), show that the total proper motion of the group must be very small.

COMAS SOLA⁴¹ considers that he has found considerable proper motions in Messier 11 at a measuring in the stereocomparator of plates with an epochal difference of only 3 years, taken with an instrument of only 16 cm's aperture and a focal length of 80 cm. BARNARD¹⁶ later has stated that he has not been able to establish any proper motions at all in this group out of his micrometrical measurings with the great Yerkes refractor extended over several years. As he has, besides, examined two plates with an epochal difference of 22 years in the stereocomparator without finding any proper motion of the stars in the cluster, it might be considered as established, supposing that the background and the cluster have no common motion, that the total displacement of this well-examined group must be very small.

Summing up the preceeding resume of the observations that may give information regarding the value of the proper motions of the globular clusters, it may be said that these certainly are considerably below the value $0''.01$ per year. As the mean of the total radial velocities for 10 globular clusters is 150 km, we obtain, when

$$\mu_m < 0''.01$$

and consequently:

$$\tau_m \leq 0''.01$$

according to (2):

$$\pi_m \leq 0''.00032.$$

According to CHARLIER the mean parallax for these globular clusters is $0''.0102$, from which follows:

$$\varepsilon' < 0.03.$$

SHAPLEY's parallaxes give, as a mean value for the same clusters, $\pi_m = 0''.000063$.

An acceptance of CHARLIER's parallaxes for globular clusters would give a τ_m component of the proper motion to $0''.3$, from which follows $100 \mu > 30''$, which disagrees with the results that have been dealt with above regarding the value of the total proper motion of the globular clusters.

Another way of determining distances for globular clusters is given by the method employed by KAPTEYN to find with the aid of the luminosity-law the mean parallax of a group of stars that can be supposed to be all at the same distance. If the stars brighter than m_0 have the mean magnitude $\bar{m}$, KAPTEYN¹⁰⁰ finds the relation:

$$\bar{m} = \frac{\int_{-\infty}^{m_0} m e^{-h^2(m+5-K+5 \log \pi)^2} dm}{\int_{-\infty}^{m_0} e^{-h^2(m+5-K+5 \log \pi)^2} dm}$$

or:

$$\bar{m} = K - 5 - 5 \log \pi - \frac{1}{2h} \frac{e^{-h^2(m_0 - K + 5 + 5 \log \pi)^2}}{\int_{-\infty}^{h(m_0 - K + 5 + 5 \log \pi)} e^{-z^2} dz} \quad (3)$$

Here it is supposed, that the luminosity-law has the form given by KAPTEYN:

$$\frac{d\varphi(M)}{dM} = \frac{h}{V_{3,1415\dots}} e^{h^2(M-K)^2}. \quad (4)$$

Concerning M 3 VON ZEIPEL has found another form for the luminosity-law:

$$\varphi(M) = B(m) \quad (5)$$

where $B(m)$ means the number of stars brighter than m^{259} .

From data that have been at my disposal, I have examined the form of the luminosity-law in M 3, M 11, M 67 and N. G. C. 1647⁸⁵, and found that they give a law of the same form as the one VON ZEIPEL has found²⁵⁹. However, these data allow only a determination of the *form* of the luminosity-curve, but leave a constant dependent on the unknown parallax undetermined, which causes that one cannot enter this law into KAPTEYN's integral equation without making special suppositions about the distance or the absolute magnitude of any globular cluster. Hence I have started from KAPTEYN's luminosity-law for all spectral classes, given in Groningen Publ.¹⁰² and A. J. 24¹⁰³, of the following form:

$$\frac{d\varphi(M)}{dM} = \frac{0,248}{V_{3,14\dots}} e^{-0,248^2(M-9,5)^2}. \quad (6)$$

Just as these lines were being written, VAN SCHOUTEN's work came under my notice. In a great investigation he has by means of KAPTEYN's method examined the luminosity-law and discussed the investigations hitherto made in regard to this question¹⁷⁹. He finds as the main result a confirmation of the form given by KAPTEYN in Groningen Publ. No. 11 for the dependence of the luminosity on the absolute magnitude. In a more recent work¹⁸⁰ he uses this law reduced to the visual Harvard scale and with the aid of calculations from various photometric catalogues examines the parallaxes for a number of clusters, among which the globulars M 3, 5, 13 and 11. A slight uncertainty is brought into this investigation by the fact that, at the reduction of the photographic measurements to visual ones, the author has, I suppose, not been able to correct for the spectral types of the stars. The, probably, too high mean value of the parallaxes obtained by VAN SCHOUTEN is, nevertheless, nearly 28 times lower than that of CHARLIER, but only 8 times higher than that of SHAPLEY, which speaks in favour of the latter parallax-system as being the more correct one.

By using KAPTEYN's luminosity-law (6) and integral equation (3), the following values are obtained for the parallaxes of M 3 and 13, if, for both of them, SHAPLEY's

data for the photo-visual magnitudes are used as a basis, and if only those stars are used that are at a distance $\geq 2'$ from the centre of the clusters:

M_0	$\pi_{M_{13}}$
14,99	0'',000290
	π_{M_3}
15,99	0'',000154

Owing to the fact that it has perhaps not been possible for SHAPLEY to survey all the stars down to the limiting magnitude 15,8 and 17,0 respectively, I have chosen the above given limits for m_0 . For m_0 13,99 and 14,99, one would obtain the values 0'',000100 and 0'',000200 for the parallaxes. The limits 15,79 and 16,99 give higher parallax-values.

The most certain values of the parallax in conformity to the above process are obtained if one has determined the luminosity curves for different spectral classes, and knows the photometrical magnitude of the stars within a certain spectral class up to the magnitude m_0 . Generally it has not yet been possible to determine the luminosity-law for the different spectral classes, but for B_0-B_5 KAPTEYN¹⁰⁰ has obtained a no doubt good determination of the luminosity-curve. He has also determined the luminosity-curve for the B_5-B_0 and A_0-A_9 stars¹⁰¹.

KAPTEYN's frequency-curve for B_0-B_5 has the following form:

$$\frac{0,409}{\sqrt{3,14\dots}} e^{-0,409^2 (M-0,885)^2} \quad (7)$$

If we suppose that SHAPLEY's colour-classes correspond to spectral classes, and that, consequently, his stars designed b_0-b_5 belong to the spectral types B_0-B_5 , we obtain in accordance with (3) and (7):

$$\begin{aligned} \pi_{M_3} &= 0'',000043, \\ \pi_{M_{13}} &= 0'',000106. \end{aligned}$$

This determination is very likely the most certain of the determinations made by means of the luminosity-law, and the double weight ascribed to these values seems to be well justified. We are now going to use the frequency-curve for B_5-B_9 , which has the form:

$$\frac{0,80}{\sqrt{3,14\dots}} e^{-0,80^2 (M-2,5)^2}.$$

From SHAPLEY's data is obtained:

$$\begin{aligned} \pi_{M_3} &= 0'',000210, \\ \pi_{M_{13}} &= 0'',000190. \end{aligned}$$

Finally, putting the frequency-curve for $A_0 - A_9$:

$$\frac{0,80}{\sqrt{3,14}} e^{-\frac{0,80^2}{2} (M-3,4)^2}$$

in (3), and solving the equation, we get:

$$\pi_{M_3} = 0'',000240,$$

$$\pi_{M_{13}} = 0'',000267.$$

Thus we have obtained the following parallax-determinations for the two globular clusters:

π_{M_3}	Weight	$\pi_{M_{13}}$	Weight
$0'',000043$	2	$0'',000106$	2
210	1	190	1
240	1	267	1
290	1	154	1
$0'',000163$		$0'',000164$	

from which we obtain:

$$\varepsilon' = 0,009.$$

Besides for M 3 and 13 there are photometric data for M 11 and 37. M 11 is examined by SHAPLEY, who has, by the same method as he has used regarding the former star-groups, determined the colour-indices for 364 stars. In spite of the high precision, which, no doubt, distinguishes SHAPLEY's data also in this case, I have not found it advisable to calculate ε' from them, as M 11 seems through its unmistakeable connection with the star cloud in Scutum Sobiesii to occupy a somewhat peculiar position among globular clusters. SHAPLEY says, that the density of the stars fainter than magnitudes 14,0 or 14,5 is little, if any, greater in the cluster than outside, and there is no evidence on any of the photographs of the existence of the highly concentrated background of faint stars typical of truly globular clusters. As a separating of the background cannot be done with the observations in hand, I have not taken any consideration to the value for $\varepsilon' = 0,027$, which follows from the mean of the parallax $0'',000400$, which has come out of 4 determinations.

For M 37 there exist measurements of photographic magnitudes, performed by NORDLUND, GIEBELER and JOY, whereby the scales have been fixed by aid of VON ZEIPPEL's visual magnitudes determined by means of a Zöllner photometer. As, consequently, the magnitudes given by the first-mentioned authors can be regarded neither as correct visual ones nor as photographic ones, I have not desired to delay with the determination of the scale-value from this cluster, until the appearance of SHAPLEY's and VON ZEIPPEL's photometric catalogues over M 37, that are now being worked out.

Joy's data give $m_0 = 13.0$; $\bar{m} = 12.13$, from which follows:

$$\pi_{M_{97}} = 0''.000550$$

and

$$\varepsilon' = 0.017.$$

The leaving out of the two values for ε' , that were obtained from M 11 and M 37, does not alter anything as to the final result. For if we considered all 4 clusters, we should obtain:

M 3	$\varepsilon' = 0.010$
M 13	009
M 11	27
M 37	17
	0.016,

a value of the same order as the one determined from M 3 and M 13.

Another reason in excluding M 11 and M 37 is to be found in the circumstance that no data exist on the frequency of the $B_0 - B_5$ stars, and that, accordingly, the parallax cannot in this case be derived from the luminosity-law with the same certainty as has been possible for the two other clusters.

Under the supposition that the luminosity-law has the same form in clusters as in the stellar system, we can, starting from the form for $B(m)$ given by photometric measurements, deduce by the graphic method parallaxes for clusters by removing the curve for $B(m)$, until it coincides with $\varphi(M)$, which, as we have supposed, has for all spectral classes the form corrected and established by VAN SCHOUTEN, and for the classes $B_0 - B_5$, $B_5 - B_9$, $A_0 - A_9$ the form, deduced from the corresponding frequency-curves given in (7)–(9).

We obtain by this method:

π_{M_3}	$\pi_{M_{13}}$	
0'',000022	0'',000045	} From all colours
32	22	
40	130	} From the colour-classes $b_0 - b_5$
96	240	
174		
		} $b_5 - b_9$
		} $a_0 - a_9$
0'',000075	0'',000109	

which gives:

$$\varepsilon' = 0.0053.$$

If we employ VAN SCHOUTEN's method, which also implies a comparison between the form of the luminosity-curves — with the difference that he does not compare the curves entire, but section by section — we obtain:

$$\pi_3 = 0''.000250,$$

$$\pi_{13} = 0''.000050,$$

from which follows an ϵ' value about double the size of the one given above.

A third way of examining the distance to globular clusters is attainable by the data we possess regarding the absolute magnitude of different spectral classes. In his treatise CHARLIER holds the view, that it is not certain that SHAPLEY's negative colour-indices in globular clusters prove that the stars in question are *B* stars. Even if we could, with regard to the intimate connection between spectral class and colour-index for the earlier spectral classes, consider it justifiable to accept SHAPLEY's opinion, that his colour-classes $b_0 - b_9$ correspond to the spectral types $B_0 - B_9$, we shall, nevertheless, presume that they belong instead to the types $A_0 - A_9$. CHARLIER, for *A* stars $m < 6.0$, gives $M = +0.47$, and for parallax stars $M = +3.04$. The weighted mean of these values is $+0.57$. If we suppose that the absolute magnitude has on an average this value also in globular clusters, we obtain the following hypothetical parallaxes:

	π	Material
M 3	$\leq 0''.000082$	$b_0 - f_0$
M 13	122	$b_0 - f_0$
(M 11	0.000216	$< f_0$)

The investigations concerning the total spectra of globular clusters have given the result that the mean type is *F*. We presume that the stars in the above-mentioned clusters, which were determined by SHAPLEY, are on an average of the type *F*, and now find from CHARLIER's data³⁹ for the absolute magnitude:

	π	
M 3	$\leq 0''.000290$	All colours.
M 13	430	» »
(M 11	0.000752	» »)

If we use the formula given by RUSSELL¹⁷¹, valid for clusters:

$$M = 0.6 + 2.1 (Sp. - 2) \quad (8)$$

where *Sp.* means spectral index ($B=1$, $A=2$ etc.), and *M* the absolute magnitude, we obtain:

	π from $b_0 - f_0$	π from all stars
M 3	$\leq 0''.000083$	$\leq 0''.000263$
M 13	124	398
(M 11	0.000219	0.000692)

If we take the mean of these determinations, we obtain the following new determination of ε'

$$\varepsilon' \leq 0,012.$$

Considering that RUSSELL's formula takes into account the considerable dispersion in M appearing in the spectral class F on account of the division, beginning there, between the giant and dwarf series, and with regard to the circumstance that at the above-mentioned calculations also negative colour-indices amounting to $-0,4^m$ have been attributed to A -stars, we must hold the opinion that the value obtained for ε' is an upper limit. This conclusion is also confirmed by the circumstance that SHAPLEY's colour-indices, reduced to the Harvard North Polar sequence, would obtain as great negative values as $-0,7^m$, a circumstance showing that SHAPLEY's colour-indices are not possibly due to a systematic error in the Mount Wilson scale, on which his investigations are based.

Thus we have obtained the following determinations of ε' :

- $\varepsilon' < 0,03$ (From the total motions)
- $\varepsilon' \leq 0,012$ (From RUSSELL's & CHARLIER's data for the absolute magnitudes)
- $\varepsilon' = 0,009$ (From the luminosity-curves according to (3))
- $\varepsilon' = 0,005$ (From the luminosity-law according to graphical construction)
- $\varepsilon' \leq 0,014$

In view of the fact that the first value indicates only an upper limit, and in reality is certainly lower, it will be justified to round off ε' downwards and suppose as a definitive value:

$$\varepsilon' = 0,01.$$

We consequently obtain the system of hypothetic parallaxes, which is given in the adjoining table. The concordance with SHAPLEY's parallaxes is particularly good, and even though future determinations will somewhat modify the results, it will, nevertheless, be considered as shown that CHARLIER's parallax-system cannot be accepted without coming into collision with our present knowledge as to the motions of globular clusters, as to the distribution of apparent magnitudes in the objects that have been previously examined, and with the conceptions of the absolute magnitudes of the nearer as well as of the remoter stars.

By using the luminosity-law I have obtained smaller parallax-values than VAN SCHOUTEN, from whose parallax-estimations ε' would obtain the value 0,03. The data given by SHAPLEY as regards the estimation of the parallax of the Hercules cluster, at which he has also used KAPTEYN's luminosity-curves and obtained smaller parallax values than mine, give a value for ε' of only 0,0013. Since VAN SCHOUTEN's treatise has not been accessible to me, and SHAPLEY does not mention the form of the luminosity-law, on which he has based his determinations, but gives only the final results, it

has not been possible for me to examine any closer the reasons for the divergent results. Even by using VAN SCHOUTEN's parallaxes we should, nevertheless, obtain values which would be only three times as great as the ones given in the table. Even this circumstance would imply an essential modification of CHARLIER's results.

Table II.

N. G. C.	π_{Sh}	π_{Lk}	m_H	m_{25}	$m_H - m_{25}$	π_H
104	0'',000148	0'',000374	—	—	—	—
288	53	98	—	—	—	—
362	66	98	—	—	—	—
1261	39	43	—	—	—	—
1851	58	71	—	—	—	—
1904	39	40	8,0	15,3	7 ^m ,3	—
2099	—	296	6,7	(14,1)	—	0'',000076
2298	41	35	—	—	—	—
2808	59	68	—	—	—	—
3201	68	103	—	—	—	—
4147	19	14	9,4	16,6	7,2	—
4372	88	152	—	—	—	—
4590	62	76	8,2	(15,6)	—	0,000038
4833	61	72	—	—	—	—
5024	53	67	7,8	15,1	7,3	—
5139	153	400	—	12,3	—	—
5272	72	125	6,6	14,2	7,6	—
5286	51	77	—	—	—	—
5634	33	22	9,7	(17,1)	—	0,000019
5897	67	86	10,2	(17,6)	—	0,000015
5904	80	134	6,7	14,0	7,3	—
5927	—	93	—	—	—	—
5986	48	53	—	—	—	—
6093	50	71	7,8	14,9	7,1	—
6101	47	49	—	—	—	—
6121	88	173	6,8	13,8	7,0	—
6144	41	50	—	—	—	—
6171	62	68	9,0	(16,4)	—	0,000026
6205	90	150	5,8	13,7	7,9	—
6218	81	112	6,8	14,0	7,2	—
6229	23	14	8,6	16,2	7,6	—
6235	20	15	9,7	(17,1)	—	0,000019
6254	83	102	6,9	14,1	7,2	—
6266	66	70	7,0	(14,4)	—	0,000066
6273	63	64	6,8	(14,2)	—	0,000072
6284	27	22	9,5	(16,9)	—	0,000021
6287	23	21	9,2	(16,6)	—	0,000024
6293	38	35	8,5	(15,9)	—	0'',000023
6304	0'',000031	0'',000024	—	—	—	—

N. G. C.	π_{S_h}	π_{L_k}	m_H	m_{25}	$m_H - m_{25}$	π_H
6316	0'',000019	0'',000013	—	—	—	—
6333	40	44	7,3	15,6	8,3	—
6341	81	97	6,2	13,9	7,7	—
6356	26	23	8,5	17,2	8,7	—
6362	77	99	—	—	—	—
6388	36	31	—	—	—	—
6397	120	232	—	—	—	—
6402	43	54	7,8	15,4	7,6	—
6441	22	19	—	—	—	—
6522	—	19	—	—	—	—
6528	—	9	—	—	—	—
6541	68	90	—	—	—	—
6553	—	22	—	—	—	—
6558	—	13	—	—	—	—
6569	—	22	—	—	—	—
6584	38	31	—	—	—	—
6624	35	28	—	—	—	—
6626	54	52	7,9	14,9	7,0	—
6637	47	39	9,0	(16,4)	—	0'',000026
6638	29	19	9,4	16,2	6,8	—
6642	26	28	—	16,1	—	—
6652	32	23	—	—	—	—
6656	118	185	6,2	13,1	6,9	—
6681	55	46	9,5	(16,9)	—	0,000021
6705	—	155	6,8	(14,2)	—	0,000072
6712	32	35	8,9	16,1	7,2	—
6715	62	46	—	—	—	—
6723	79	94	—	—	—	—
6752	114	208	—	—	—	—
6779	40	32	8,3	15,3	7,0	—
6809	100	161	—	—	—	—
6864	22	24	8,0	17,1	9,1	—
6934	30	27	9,0	15,8	6,8	—
6981	34	30	9,5	15,9	6,4	—
7006	15	10	9,7	(17,1)	—	0'',000019
7078	68	84	6,2	14,3	8,1	—
7089	64	99	6,7	14,6	7,9	—
7099	0'',000058	77	8,5	14,6	6 ^m ,1	—
7214	—	0'',000004	—	—	—	—

The total magnitudes of globular clusters, estimated with great care by HOLETSCHEK⁹¹⁻⁹³, seem to furnish a proper measure for their relative parallaxes. We find, according to the table, that the difference between SHAPLEY's value for the mean magnitude of the 25 brightest stars in a cluster, m_{25} , and HOLETSCHEK's value for the magnitude m_H , has a conspicuous constant value. On an average is:

$$m_{25} - m_H = + 7,40^m. \quad (9)$$

With the use of this law the values for m_{25} , put in parentheses, have been calculated, and out of these values the parallaxes π_H have been obtained according to SHAPLEY's method.

Of the 15 parallaxes obtained in this way, 8 agree well with SHAPLEY's values, and in regard to N. G. C. 2099 and 6705 the concordance with parallax-values calculated after SHAPLEY's principles would certainly also be a very good one. The remaining 5 values are of the same order of magnitude as π_{sh} , and the difference between π_{sh} and π_H is considerable only as regards N. G. C. 5897. KRITZINGER¹⁰⁸ gives a lower value for the total magnitude, from which would follow $\pi_H = 0'',000020$. How to account for the remaining difference, I will for the present leave out of consideration.

The great value of observations of the total magnitudes is shown by the calculated π_H . It is very desirable that the determination photographically of the total magnitudes of clusters, for instance by means of extra-focal photometry, would be undertaken. A step in that direction has been made by HERTZSPRUNG⁸⁸, and it is to be hoped that his works on this subject will be continued.

Otherwise there would present themselves several ways of determining *relative* parallaxes for globulars. The strong correlation shown by CHARLIER³⁹ between the number of stars and the diameter in globular clusters, indicates a way of obtaining relative parallaxes, and determinations of the apparent average distance between the stars for instance by counting the number of stars in a certain area on a cluster plate, another. In view of the scantiness of the material at hand in these cases we shall nevertheless not at present occupy us with the determinations of relative parallaxes by means of these methods.

II. Determinations of the Solar Apex from the Velocities of Clusters and Nebulæ.

SLIPHER's 10 radial velocities in Table III for globular clusters give us a possibility of determining the Apex of the solar motions in its relation to the former. Thereby one may be able to test whether these objects are far away or in our immediate neighbourhood. For in the latter case the value for the Apex ought not to differ very much from the one adopted earlier, nor should the speed of the sun's motion turn out to be of another order. The small number of velocities measured, naturally causes that the Apex that is being determined by their aid, cannot possess any high degree of accuracy. Yet we will here put in mind the fact that HERSCHEL in 1783 with only 13 stars obtained an Apex-value that differs only very little from the one now commonly adopted.

We start at the determinations from the equation $K + \cos \alpha \cos \delta X + \sin \alpha \cos \delta Y + \sin \delta Z = V$, where K means a systematic correction to the radial velocity V and X , Y and Z are defined by the equation:

$$\begin{aligned} V_0^2 &= X^2 + Y^2 + Z^2, \\ \operatorname{tg} A &= \frac{Y}{X}, \\ \sin D &= \frac{Z}{V_0}, \end{aligned} \quad (10)$$

where A and D signify the right ascension and declination for the Apex and V_0 the speed of the solar motion.

Concerning the appropriateness of this method to determine Apex we beg to call attention to the fact that, according to MANSON's researches (A. J. 713), »it is not necessary to assume that the radial motions are principally due to the motion of the sun or that the exponential Law of Error is followed by the peculiar motions of stars used in the solution».

Table III.

N. G. C.	α_{1900}	δ_{1900}	V
5024	197,°0	+18,°7	-170 km' sec
5272	204,4	+28,9	-125
5904	228,4	+ 2,5	+ 10
6205	249,5	+36,7	-300
6333	258,3	-18,4	+225
6341	258,5	+43,3	-160
6626	274,6	-24,9	± 0
6934	307,3	+ 7,1	-410
7078	321,3	+11,7	- 95
7089	322,1	- 1,3	- 10

We obtain, according to the equation given above the following equations of condition, where 1000 km. is taken as a unit for the radial motions:

$$\begin{aligned} K - 0,906 X - 0,277 Y + 0,321 Z &= -0,170 \\ K - 0,797 X - 0,362 Y + 0,483 Z &= -0,125 \\ K - 0,663 X - 0,747 Y + 0,044 Z &= +0,010 \\ K - 0,281 X - 0,751 Y + 0,598 Z &= -0,300 \\ K - 0,192 X - 0,929 Y - 0,316 Z &= +0,225 \\ K - 0,145 X - 0,713 Y + 0,686 Z &= -0,160 \\ K + 0,073 X - 0,904 Y - 0,421 Z &= 0,000 \end{aligned}$$

$$\begin{aligned}
K + 0,601 X - 0,789 Y + 0,124 Z &= -0,410 \\
K + 0,764 &- 0,612 &+ 0,203 &- 0,095 \\
K + 0,789 &- 0,614 &- 0,023 &- 0,010
\end{aligned}$$

which give the normal equations:

$$\begin{aligned}
10,000 K - 6,698 Y - 0,757 X + 1,699 Z &= -1,025 \\
- 6,698 &+ 4,893 &+ 0,035 &- 0,768 &+ 0,603 \\
- 0,757 &+ 0,035 &+ 3,606 &- 0,730 &- 0,016 \\
+ 1,699 &- 0,768 &- 0,730 &+ 1,501 &- 0,545
\end{aligned}$$

from which follow the elimination equations:

$$\begin{aligned}
1,0000 K - 0,6698 Y - 0,0757 X + 0,1699 Z &= -0,1035 \\
1,0000 Y - 1,1597 X + 0,9091 Z &= -0,2211 \\
1,0000 X - 0,0573 Z &= -0,0660 \\
1,0000 Z &= -0,3440
\end{aligned}$$

which give the following values for the unknown quantities:

$$\begin{aligned}
X &= -0,086 & Z &= -0,344 & V_0 &= 0,3547 \\
Y &= -0,008 & K &= -0,057 & A &= + 5^\circ \\
D &= + 76^\circ
\end{aligned}$$

If we put $K=0$, we obtain:

$$\begin{aligned}
1,0000 Y + 0,0071 X - 0,1570 Z &= + 0,1232 \\
1,0000 X - 0,2011 Z &= -0,0055 \\
1,0000 Z &= -0,3673 \\
X &= -0,079 & V_0 &= -0,381 \\
Y &= + 0,066 & A &= -40^\circ \\
Z &= -0,367 & D &= + 74^\circ \\
E_x &= \pm 0,080 & E_V &= \pm 0,154 \text{ (m. e.)} \\
E_y &= \pm 0,069 & E_A &= \pm 59^\circ \\
E_z &= \pm 0,130 & E_D &= \pm 44^\circ
\end{aligned}$$

Residuals.

V	$K = -57$ v	$K = 0$ v	V	$K = -57$ v	$K = 0$ v
-170 km	+ 84 km	+ 16 km	-160 km	-114 km	-128 km
-125	- 25	- 14	0	+102	+ 89
+ 10	- 18	- 22	-410	+265	+265
-300	- 68	+ 52	- 95	- 92	- 80
+225	-148	-155	- 10	-101	- 84

With regard to the great uncertainty in the unknown quantities in the former case it seems not to be justified to introduce K into the equations of condition and it will therefore be appropriate to consider the latter solution as the definitive one.

It is evident that the Apex-value thus found has nothing to do with the solar Apex commonly adopted. This circumstance gets its natural explanation when we adopt SHAPLEY's opinion, that we are surrounded by an ellipsoidal stellar cluster, whose plane of symmetry inclines only 12° towards the galactic plane. This *local cluster* contains the majority of the B and A stars known, and will possibly be identified with CHARLIER's »Galaxy» and GOULD's belt of bright stars. Thus it becomes possible also to understand the origin of CHARLIER's great parallaxes for clusters. From his supposition, that the Helium-stars are phenomena characteristic of the Galaxy, he has been led to quite too small dimensions for our stellar systems. The B -stars are evidently phenomena appearing in cluster formations. In the new Draper-Catalogue we thus find 26 B_0-B_3 stars in and close up to the galactic cluster h & χ Persei. HERTZSPRUNG's and the author's measurements of effective wave-lengths in N. G. C. 1647 show that the majority of the stars belonging to this cluster must be considered as A and F stars, but that few, if any, B stars are to be found in this group. Measurements by BERGSTRAND not yet completely finished over effective wave-lengths in the star cloud in Cygnus also show many A stars but probably very few B stars.

With the adoption of the local cluster theory^{198,199} it also follows that the solar Apex should be dependent on the average distance to the objects employed in its determination. Should bright stars of earlier types be employed, we obtain a solar Apex with respect to stars within the cluster. Again, should we employ remote stars we ought to obtain an Apex for the solar motion with respect to the galactic system. Our Apex calculated above should consequently best correspond to the motion and velocity of the local cluster with respect to the globular cluster system.

It is interesting to see how the declination for the solar Apex increases, the fainter the stars employed are. Thus DYSON and THACKERY⁵⁸ found:

A	D	$Mag.$ $m \quad m$
245°	+ 16°	1.0 — 4.9
268	+ 27	5.0 — 5.9
278	+ 33	6.0 — 6.9
280	+ 38,5	7.0 — 7.9
272	+ 43	8.0 — 8.9

COMSTOCK⁴² found:

280	+ 58	8.6
288	+ 71	11.1

The concordance between the Apex-value derived from cluster motions and that of COMSTOCK is striking, and perhaps more than a contingency.

It would be of great interest to see whether the velocity V_0 increases when faint stars are involved in the determination of Apex. On my request, Mr. K. LITTMARCK has determined the solar Apex from those stars with considerable radial velocities and small proper motions, compiled by PERRINE in A. J. 30¹⁵³.

He finds from the 71 equations of condition, assuming $K=0$, the following normal equations:

$$\begin{aligned} + 16,119 X + 2,527 Y + 2,957 Z &= + 0,899 \\ + 2,527 \quad + 12,639 \quad + 1,954 &= - 1,827 \\ + 2,957 \quad + 1,954 \quad + 29,194 &= - 5,071 \end{aligned}$$

From this he obtains:

$$\begin{aligned} A &= 310^{\circ},6 \quad D = + 50^{\circ},0 \\ V_0 &= - 27,7 \text{ km./sec.} \end{aligned}$$

Between the Apex calculated from the radial motions of globular clusters and the one found by WIRTZ²³⁰ from the motions of spiral nebulae, there is also a remarkable concordance prevailing. The directions of the solar motion coincides closely. The speed of the sun's motion is, it is true, double the size in the latter case than in the former, but, nevertheless, of the same order of magnitude. In Table IV¹ I have compiled all material that has been at my disposal regarding the radial velocities of the spiral nebulae. We find, that the velocities have been determined with tolerably great certainty, regard having been taken to their considerable magnitude. Besides for 18 spirals the radial motions are also determined for the great and the small Magellanic clouds. The remarkable concordance I have found between the structure of the great Magellanic cloud and the nebula 4449, classified as a spiral one (HINKS, WOLF), and with a spectrum typical of the spiral nebulae, causes, that we must in reality, perhaps, consider the Magellanic clouds as the nearest spiral nebulae, Pl. 1. This circumstance may perhaps justify my employing these objects at our Apex-determinations. Owing to the fact that recent spectrographic researches have shown that it is difficult to draw a definitive limit between planetaries and spirals, because the latter often show also the spectrum typical of the *proper* nebulae, and planetary nebulae sometimes show spiral structure¹⁷⁰ (N. G. C. 1514), I have considered it appropriate to employ the four planetaries, possessing radial velocities comparable in size with those of spirals. The predominance of positive radial velocities shows the justifiableness of taking in the constant K in the equations of condition, but the latter have also been solved with the hypothesis $K=0$.

According to the equation (10) we obtain the following 24 equations of condition for the radial velocities included in the Table IV.

¹ WOLF has determined a number of radial velocities for spiral nebulae, but only stated the direction of the motions, not their size. He finds thus:

N. G. C. 224 —	N. G. C. 3031 +
598 —	4194 +
6853 —	4826 +

Table IV. Radial velocities employed at the Apex determinations.

No.	N. G. C.	α_{1900}	δ_{1900}	V observed	Observer	V adopted
1	7331	338°,1	+33°,9	+ 300	SLIPHER	+ 300 km./sec.
2	224	9,3	+40,7	- 329	PEASE & ADAMS	
				297	»	
				300	SLIPHER	
				350	WOLF	
				- 304	WRIGHT	-316 ± 10
3	221	9,3	+40,3	- 300	SLIPHER	- 300
4	598	22,0	+30,2	- 278	PEASE	- 278
				- 70	»	
5	1023	38,5	+38,6	+ 200	SLIPHER	+ 200
6	1068	39,4	- 0,4	+ 910	WRIGHT	
				765	PEASE	
				1060	SLIPHER	
				1150	»	
				1080	»	
				+1135	»	+1020 ± 60
7	1700	73,0	+ 5,0	+high	PEASE	+ 500
8	3031	146,8	+69,5	+small	SLIPHER	+ 100
9	3115	150,0	- 7,2	+ 400	»	+ 400
10	3379	160,7	+13,1	+ 830	PEASE & SANFORD	
				860	»	
				+ 780	SLIPHER	+ 825
11	3627	168,8	+13,5	+ 500	»	+ 500
12	4151	181,4	+40,0	+940 ± 40	CAMPBELL	+940 ± 40
13	4565	187,9	+26,5	+1000	SLIPHER	+1000
14	4594	188,7	-11,0	+1180	PEASE	
				+1100	SLIPHER	+1140
15	4736	191,5	+41,7	+ 200	»	+ 200
16	4826	193,0	+22,2	+small	»	+ 100
17	5194	201,4	+47,7	±small	»	0
18	5866	225,9	+56,2	+ 600	»	+ 600
19	1644	16,5	-73,7	+ 158	WILSON	+ 158
	1714			+ 301		
20	1743	80,0	-68,8	+ 254	WILSON	+ 275
	2070			+ 276		
	2111			+ 268		
21	5873	226,6	-37,7	- 133	MOORE	- 133
22	6644	276,5	-25,2	+ 202	»	+ 202
23	IC _{II} 4732	277,0	-22,7	- 136	»	- 136
24	IC _{II} 4846	287,8	-19,2	+ 165	CAMPBELL & HAYNES	+ 165

Table V. *Equations of condition.*

Spirals.	1	$K + 0,770$	$X - 0,310$	$Y + 0,558$	$Z = + 0,300$
	2	$K + 0,748$	$+ 0,123$	$+ 0,652$	$- 0,316$
	3	$K + 0,753$	$+ 0,123$	$+ 0,647$	$- 0,300$
	4	$K + 0,801$	$+ 0,324$	$+ 0,503$	$- 0,278$
	5	$K + 0,612$	$+ 0,486$	$+ 0,624$	$+ 0,200$
	6	$K + 0,773$	$+ 0,635$	$- 0,007$	$+ 1,020$
	7	$K + 0,291$	$+ 0,953$	$+ 0,087$	$+ 0,500$
	8	$K - 0,293$	$+ 0,192$	$+ 0,937$	$+ 0,100$
	9	$K - 0,859$	$+ 0,496$	$- 0,125$	$+ 0,400$
	10	$K - 0,919$	$+ 0,322$	$+ 0,227$	$+ 0,825$
	11	$K - 0,954$	$+ 0,189$	$+ 0,234$	$+ 0,500$
	12	$K - 0,766$	$- 0,019$	$+ 0,643$	$+ 0,940$
	13	$K - 0,866$	$- 0,123$	$+ 0,446$	$+ 1,000$
	14	$K - 0,970$	$- 0,149$	$- 0,191$	$+ 1,140$
	15	$K - 0,732$	$- 0,149$	$+ 0,665$	$+ 0,200$
	16	$K - 0,902$	$- 0,208$	$+ 0,378$	$+ 0,100$
	17	$K - 0,627$	$- 0,246$	$+ 0,740$	$0,000$
	18	$K - 0,387$	$- 0,399$	$+ 0,831$	$+ 0,600$
Magellanic clouds.	19	$K + 0,269$	$+ 0,080$	$- 0,960$	$+ 0,158$
	20	$K + 0,063$	$+ 0,356$	$- 0,932$	$+ 0,275$
Planetaries.	21	$K - 0,544$	$- 0,575$	$- 0,612$	$- 0,133$
	22	$K + 0,102$	$- 0,899$	$- 0,426$	$+ 0,202$
	23	$K + 0,112$	$- 0,916$	$- 0,386$	$- 0,136$
	24	$K + 0,289$	$- 0,899$	$- 0,329$	$+ 0,165$

If all objects are included and the normal equations are formed, we obtain:

$$\begin{aligned}
 24,000 K - 3,256 X - 0,613 Y + 4,204 Z &= + 7,462 \\
 - 3,256 &+ 10,684 &+ 1,089 &- 0,872 &= - 4,000 \\
 - 0,613 &+ 1,089 &+ 5,442 &+ 1,012 &= + 0,922 \\
 + 4,204 &- 0,872 &+ 1,012 &+ 7,882 &= + 1,224 \\
 1,0000 K - 0,1357 X + 0,1752 Z - 0,0255 Y &= + 0,3109 \\
 1,0000 X - 0,0295 Z + 0,0982 Y &= - 0,2918 \\
 1,0000 Z + 0,1610 Y &= - 0,0240 \\
 1,0000 Y &= + 0,2787
 \end{aligned}$$

$$\begin{aligned}
X &= -0,321 & Z &= -0,069 \\
Y &= +0,279 & K &= +0,287 \\
V_0 &= -0,430 \pm 0,225, & A &= -34^\circ \pm 24^\circ \\
& & D &= +9^\circ \pm 24^\circ.
\end{aligned}$$

We then put $K=0$ and obtain the elimination equations:

$$\begin{aligned}
1,0000 X - 0,0816 Z + 0,1019 Y &= -0,3744 \\
1,0000 Z + 0,1410 Y &= +0,1150 \\
1,0000 Y &= +0,2324 \\
X &= -0,391 & Z &= +0,082 \\
Y &= +0,232 \\
V_0 &= -0,462 \pm 0,273, & A &= -27^\circ \pm 35^\circ \text{ (m. e.)} \\
E_0 &= \pm 0,473 & D &= -10^\circ \pm 27^\circ.
\end{aligned}$$

E_0 = mean error in one velocity with the weight = 1.

The two Apex-values differ with respect to the co-ordinates considerably from the Apex determined from the radial motions of the globular clusters, while, on the contrary, V_0 agrees well in both cases with the value found from SLIPHER's radial velocities.

Introducing the values of the unknown quantities into the equations of condition, we obtain the following residuals. The greater part of a residual surely can be ascribed to individual motions of the examined objects, and only a small part can be considered due to observational errors in the velocities.

Residuals for all objects.

$-V$	$K = +287$ v	$K = 0$ v	$-V$	$K = +287$ v	$K = 0$ v
- 300 km	- 385 km	- 627 km	-1000 km	- 493 km	- 646 km
+ 316	+ 352	+ 106	-1140	- 571	- 812
+ 300	+ 334	+ 88	- 200	+ 234	+ 106
+ 278	+ 363	+ 281	- 100	+ 393	+ 236
- 200	- 16	- 275	- 0	+ 368	+ 249
-1020	- 804	-1176	- 600	- 357	- 474
- 500	- 46	- 386	- 158	+ 131	- 323
- 100	+ 270	+ 137	- 275	+ 155	- 293
- 400	+ 310	- 41	+ 133	+ 477	+ 163
- 825	- 169	- 372	- 202	- 170	- 486
- 500	+ 130	- 64	+ 136	+ 158	- 153
- 940	- 456	- 591	- 165	- 199	- 514

If the 4 planetary nebulae are excluded but the Magellanic clouds retained in the solution, we obtain the normal equations:

$$\begin{aligned}
20,000 K - 3,215 X + 5,957 Z + 2,676 Y &= + 7,364 \\
-3,215 &+ 10,281 &- 1,024 &+ 1,231 &= - 4,126 \\
+ 5,957 &- 1,024 &+ 7,069 &- 0,373 &= + 1,231 \\
+ 2,676 &+ 1,231 &- 0,373 &+ 2,656 &= + 1,051
\end{aligned}$$

The elimination equations:

$$\begin{aligned}
1,0000 K - 0,1608 X + 0,2979 Z + 0,1338 Y &= + 0,3682 \\
1,0000 X + 0,0067 Z + 0,1701 Y &= - 0,3013 \\
1,0000 Z - 0,2189 Y &= - 0,1854 \\
1,0000 Y &= + 0,1993
\end{aligned}$$

$$\begin{aligned}
X &= - 0,336, V_0 = - 0,415 \pm 0,311 \\
Y &= + 0,199, A = - 31^\circ \pm 47^\circ \\
Z &= - 0,142, D = + 21^\circ \pm 44^\circ \\
K &= + 0,330 \\
E_0 &= \pm 0,406.
\end{aligned}$$

And, supposing $K = 0$:

$$\begin{aligned}
1,0000 X - 0,0996 Z + 0,1197 Y &= - 0,4013 \\
1,0000 Z - 0,0359 Y &= + 0,1177 \\
1,0000 Y &= + 0,6296
\end{aligned}$$

we obtain:

$$\begin{aligned}
X &= - 0,463, V_0 = - 0,794 \pm 0,354 \\
Y &= + 0,630, A = - 55^\circ \pm 21^\circ \\
Z &= + 0,140, D = - 10^\circ \pm 18^\circ \\
E_0 &= \pm 0,469
\end{aligned}$$

from which follow the residuals:

$-V$	$K = + 330$ v	$K = 0$ v	$-V$	$K = + 330$ v	$K = 0$ v
- 300 km	- 370 km	- 774 km	- 500 km	+ 156 km	+ 94 km
+ 316	+ 326	+ 138	- 940	- 448	- 507
+ 300	+ 309	+ 119	- 1000	- 459	- 605
+ 278	+ 332	+ 181	- 1140	- 487	- 812
- 200	- 68	- 90	- 200	+ 252	+ 138
- 1020	- 823	- 979	- 100	+ 438	+ 240
- 500	- 90	- 23	0	+ 387	+ 239
- 100	+ 233	+ 288	- 600	- 337	- 556
- 400	+ 336	+ 293	- 158	+ 234	- 367
- 825	- 154	- 165	- 275	+ 237	- 210

If further we exclude the Magellanic clouds, we obtain:

$$\begin{aligned}
 18,000 K - 3,547 X + 7,849 Z + 2,240 Y &= + 6,931 \\
 - 3,547 &+ 10,205 &- 0,707 &+ 1,187 &- 4,186 \\
 + 7,849 &- 0,707 &+ 5,278 &+ 0,036 &+ 1,638 \\
 + 2,240 &+ 1,187 &+ 0,036 &+ 2,523 &+ 0,940 \\
 1,0000 K - 0,1970 X + 0,4361 Z + 0,1244 Y &= + 0,3851 \\
 1,0000 X + 0,0884 Z + 0,1713 Y &= - 0,2967 \\
 1,0000 Z + 0,6087 Y &= - 0,6379 \\
 1,0000 Y &= - 0,1011 \\
 K &= + 0,659, V_0 = - 0,738 \pm 0,211 \\
 X &= - 0,218, A = + 25^\circ \pm 89^\circ \\
 Y &= - 0,101, D = + 71^\circ \pm 27^\circ \\
 Z &= - 0,698 \\
 E_0 &= \pm 0,391
 \end{aligned}$$

and if we put $K=0$, we obtain:

$$\begin{aligned}
 1,0000 X - 0,0693 Z + 0,1163 Y &= - 0,4102 \\
 1,0000 Z + 0,0226 Y &= + 0,2577 \\
 1,0000 Y &= + 0,5865 \\
 X &= - 0,461, V_0 = - 0,785 \pm 0,377 \\
 Y &= + 0,586, A = - 52^\circ \pm 23^\circ \\
 Z &= + 0,244, D = - 18^\circ \pm 25^\circ.
 \end{aligned}$$

The unknown quantities give the following residuals:

$-V$	$K = + 659$ v	$K = 0$ v	$-V$	$K = + 659$ v	$K = 0$ v
- 300 km	- 168 km	- 701 km	- 825 km	- 158 km	- 157 km
+ 316	+ 344	+ 202	- 500	+ 184	+ 108
+ 300	+ 331	+ 183	- 940	- 561	- 441
+ 278	+ 377	+ 222	- 1000	- 448	- 555
- 200	- 159	- 45	- 1140	+ 129	- 827
- 1020	- 589	- 1005	- 200	+ 169	+ 212
- 500	- 61	- 55	- 100	+ 513	+ 287
- 100	- 51	+ 377	0	+ 304	+ 326
- 400	+ 483	+ 256	- 600	- 398	- 453

This last determination gives a result almost identical to WIRTZ's. The addition of 3 new objects and a number of new radial-velocity determinations to the ones employed by WIRTZ has not essentially changed his results.

With WIRTZ one can inquire about the signification of K . The great value, with which it turns out, will hardly be ascribed to a systematic correction of the kind that has been found for the B -stars. Rather will the explanation be justified, that we have in this case to do with a systematic motion of spiral nebulæ. Yet the nature of this motion is enigmatic, owing to the fact that our determinations consider two groups of spiral nebulæ, situated in two regions of the sky opposite to one another, and consequently this would imply, that these groups are both receding from the solar system with a velocity of 660 km./sec. As a matter of course, we can not enter into the interpretation of this peculiar circumstance. Finally we will examine the two groups of nebulæ separately. If we thus take into consideration the first seven objects in Table V, we obtain the normal equations:

$$\begin{aligned}
 7,000 K + 4,748 X + 3,064 Z + 2,334 Y &= + 1,126 \\
 4,748 \quad + 3,420 \quad + 2,210 \quad + 1,271 &= + 0,602 \\
 3,064 \quad + 2,210 \quad + 1,805 \quad + 0,532 &= - 0,211 \\
 2,334 \quad + 1,271 \quad + 0,532 \quad + 1,778 &= + 0,963 \\
 1,0000 K + 0,6783 X + 0,4377 Z + 0,3334 Y &= + 0,1609 \\
 1,0000 X + 0,6633 Z - 1,5678 Y &= - 0,8141 \\
 1,0000 Z - 0,7527 Y &= - 1,5878 \\
 1,0000 Y &= - 0,3983 \\
 K &= + 1,233, V_0 = - 1,929 \pm 1,063 \\
 X &= - 0,176, A = + 66^\circ \pm 174^\circ \\
 Y &= - 0,389, D = + 77^\circ \pm 222^\circ \\
 Z &= - 1,881
 \end{aligned}$$

$K = 0$:

$$\begin{aligned}
 1,0000 X + 0,6462 Z + 0,3716 Y &= + 0,1760 \\
 1,0000 Z + 0,7666 Y &= - 1,5915 \\
 1,0000 Y &= + 0,2574 \\
 X &= + 0,982, V_0 = - 1,725 \pm 0,926 \\
 Y &= + 0,257, A = + 15^\circ \pm 40^\circ \\
 Z &= - 1,395, D = + 54^\circ \pm 55^\circ.
 \end{aligned}$$

$-V$	$K = +1233$ v	$K = 0$ v
$- 300 \text{ km}$	$- 132 \text{ km}$	$- 402 \text{ km}$
$+ 316$	$+ 143$	$+ 172$
$+ 300$	$+ 135$	$+ 168$
$+ 278$	$+ 298$	$+ 446$
$- 200$	$- 438$	$- 344$
-1020	$- 157$	$- 88$
$- 500$	$+ 147$	$- 90$

If we now consider the eleven spirals 8—18, we obtain:

$$\begin{aligned}
 11,000 K - 8,295 X + 4,785 Z - 0,094 Y &= + 5,805 \\
 - 8,295 &+ 6,785 &- 2,917 &- 0,084 &= - 4,788 \\
 + 4,785 &- 2,917 &+ 3,473 &- 0,496 &= + 1,850 \\
 - 0,094 &- 0,084 &- 0,496 &+ 0,745 &= - 0,023 \\
 1,0000 K - 0,7541 X + 0,4350 Z - 0,0085 Y &= + 0,5277 \\
 1,0000 X + 1,3037 Z - 0,2924 Y &= - 0,7754 \\
 1,0000 Z - 0,5152 Y &= - 0,2831 \\
 1,0000 Y &= - 0,2900 \\
 X &= - 0,296, V_0 = - 0,599 \pm 1,239 \\
 Y &= - 0,290, A = + 45^\circ \pm 151^\circ \\
 Z &= - 0,433, D = + 46^\circ \pm 201^\circ \\
 K &= + 0,491 \pm 0,998
 \end{aligned}$$

$K = 0$:

$$\begin{aligned}
 1,0000 X - 0,4299 Z - 0,0124 Y &= - 0,7042 \\
 1,0000 Z - 0,2397 Y &= - 0,0919 \\
 1,0000 Y &= - 0,2127 \\
 X &= - 0,771, V_0 = - 0,813 \pm 0,374 \\
 Y &= - 0,214, A = + 15^\circ \pm 39^\circ \\
 Z &= - 0,143, D = + 10^\circ \pm 26^\circ.
 \end{aligned}$$

Residuals.

$-V$	$K = +491$ v	$K = 0$ v
- 100 km	+ 15 km	- 52 km
- 400	+ 254	+ 174
- 825	- 254	- 218
- 500	+ 116	+ 162
- 940	- 495	- 439
-1000	- 405	- 356
-1140	- 237	- 332
- 200	+ 262	+ 299
- 100	+ 553	+ 585
0	+ 427	+ 428
- 600	- 239	- 338

Regarding these determinations we desire to add, that the former group gives a result differing only immaterially from the first value found for all the spirals. The introducing of K seems not to be pretty well accounted for in this case. For the group 8—18 we obtain with the other determinations not so well agreeing Apex

values, and the great value for K , which results in this case, will not have any reality. The scanty material however prevents our drawing any further conclusions.

Table VI shows the different Apex-values obtained by us.

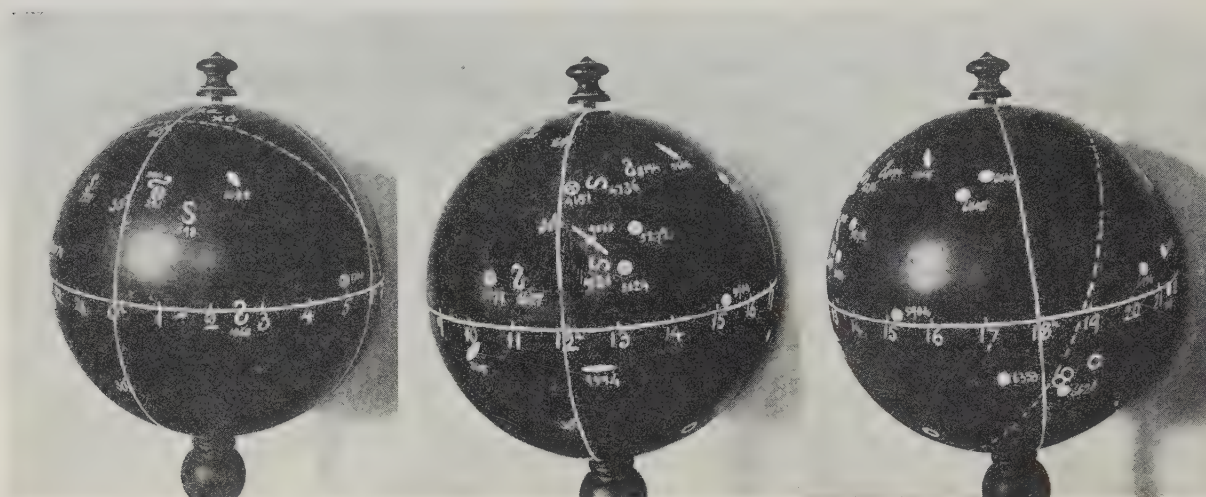


Photo Ossian Vallin.

Fig. 2. Distribution of the objects employed at the determinations of Apex.

- globular clusters
- planetaries
- ⊗ Round spirals
- Spindles
- § Spirals
- Milky Way



Photo Ossian Vallin.

Fig. 3. Distribution of the Apex-values derived from Clusters and Nebulæ.

<i>a</i> The Apex from SLIPHER's 10 velocities $K=-57$	<i>b</i> The Apex from SLIPHER's 10 velocities $K=0$
<i>c</i> The Apex derived from 18 objects $K=+659$	<i>d</i> The Apex derived from 18 objects 10 velocities $K=0$
<i>e</i> " " " " 7 " $K=+1233$	<i>f</i> " " " " 7 " $K=0$
<i>g</i> " " " " 11 " $K=+491$	<i>h</i> " " " " 11 " $K=0$
<i>i</i> " " " " 20 " $K=+330$	<i>j</i> " " " " 20 " $K=0$
<i>k</i> " " " " 24 " $K=+287$	<i>l</i> " " " " 24 " $K=0$
<i>m</i> The Apex derived by WIRTZ from nebular proper motions.	

Table VI. The Apex-values.

In this table *probable* errors are employed. V and K are expressed in km./sec.

Material employed	K	V	A	D
10 Clusters	-56 ± 107 km./sec.	-355 ± 112 km./sec.	$+5^\circ \pm 9^\circ$	$+76^\circ \pm 139^\circ$
	0	-384 ± 100	$+40 \pm 40$	$+74 \pm 30$
24 objects	$+287 \pm 57$	-430 ± 152	-34 ± 16	$+9 \pm 16$
	0	-462 ± 184	-27 ± 23	-10 ± 18
20 objects	$+330 \pm 86$	-415 ± 210	-31 ± 32	$+21 \pm 29$
	0	-794 ± 239	-55 ± 14	-10 ± 12
18 Spirals	$+659 \pm 149$	-738 ± 142	-25 ± 60	$+71 \pm 18$
	0	-785 ± 254	-52 ± 16	-18 ± 17
11 Spirals	$+491 \pm 673$	-599 ± 836	$+45 \pm 102$	$+46 \pm 136$
	0	-813 ± 252	$+15 \pm 26$	$+10 \pm 17$
7 Spirals	$+1233 \pm 717$	-1929 ± 647	$+66 \pm 118$	$+77 \pm 149$
	0	-1725 ± 455	$+15 \pm 27$	$+54 \pm 37$

It is an interesting fact, that most of the Apex-values are situated in or near the galactic plane. This is also the case with WIRTZ' Apex-value calculated from the proper motions of the 98 small nebulae, derived from the measurements made by SCHULTZ and REINMUTH²⁸⁰.

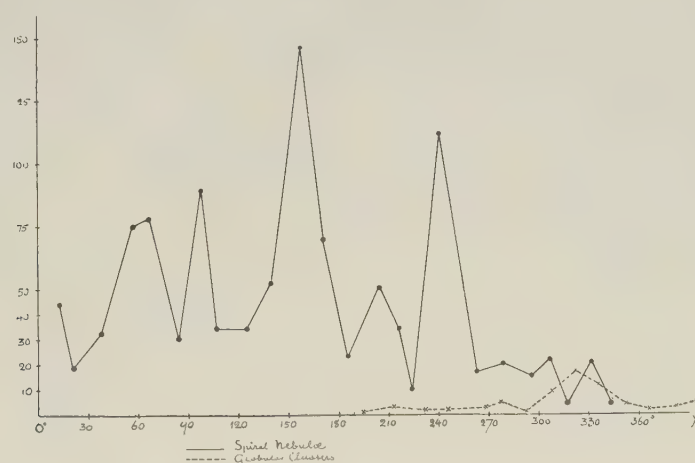


Fig. 4. Distribution of spiral nebulae and globular clusters in galactic longitude.

This fact shows what a fundamental part the Milky Way plays, not only in respect to the distribution of the spirals and globulars, but also in regard to their motions.

From the homogeneous material about the distribution of spiral nebulae communicated by FATH in A. J. 28: 86 we have calculated the galactic longitudes for the spirals and drawn the above curve, and besides the corresponding curve calculated

by SHAPLEY for the dependence on galactic longitudes of the globulars has also been dotted. We find from this that spiral nebulæ evidently concentrate in another direction than globular clusters. This circumstance does not exclude a correlation regarding the galactic distribution between the two kinds of objects but nevertheless makes the problem much more complicated.

The conclusion resulting from these Apex-values, that the spiral nebulæ and the globular clusters are in some way related objects is also confirmed by the circumstance that PERRINE has found nebulæ near a number of globular clusters¹⁴⁷. Thus he finds, in the vicinity of:

N. G. C. 7078	16	nebulæ
7089	30	»
7099	31	»

These nebulæ are elliptical or circular, and spirals in such cases when the structure has been plainly discernible. It must be more than a chance that so many nebulæ are to be found in the direction of these clusters.

With regard to the plausible connection between spiral nebulæ and globular clusters we are now going to examine the observations that might contribute to the question of the distance of the spiral nebulæ.

III. The Parallaxes of Spiral Nebulæ.

STEPHEN ALEXANDER² seems to have been the first who, taking HERSCHEL's star-gauges and Lord ROSSE's at that time strongly doubted discovery of the spiral forms of nebulæ as a starting point, expressed the opinion, that the Milky Way is a formation analogous to the spirals, which, consequently, are to be considered as remote gigantic stellar systems. The theory of the Milky Way as a spiral was expressed anew in 1870 by PROCTOR¹⁵⁹, but the hypothetic spiral he constructed as an endeavour of explaining the structure of the Milky Way has, with respect to the form, nothing to do with the structure observed in spiral nebulæ. The theory of the spiral form of the Milky Way has later been exposed by EASTON^{59, 60} in a series of investigations. He has lately succeeded in constructing satisfactorily the form of the Milky Way from the existing photographic material, supposing that it is a spiral nebula of a form corresponding to a type intermediate between M 51 and M 101⁶⁰.

The views regarding the relation of spiral nebulæ to the stellar system have varied a great deal during the last few decades. Astronomers have sometimes been inclined, in these objects, now to be found in so great a number, to see solar systems just being formed (MOULTON & CHAMBERLIN³⁰, SEE¹⁵⁷), sometimes very remote stellar systems, »Island Universes», Galaxies & C. (CURTIS⁵⁰, VERY²²⁴ and others), sometimes nebulous vortices²⁴ and, quite lately, objects certainly remote, but nevertheless at such distances that they form together with globular clusters and our stellar system a gigantic world-unit, symmetrically distributed about the galactic plane²⁰¹.

During the last few years considerable progress has been made as regards the study of the nebulae, and although it cannot be said, that any definitive fact has yet been established as to their cosmogony, we can, however, say that almost all observational facts tend to show that spiral nebulae must be considered as very remote objects. The reasons for this statement are principally the following.

1. *Direct parallax measurements.*

From what has now been published concerning attempts to measure directly parallaxes for spiral nebulae, we have compiled the following

Table VII.

Object	π	Observer
M 32	0,"0	HALL ⁷³
	$-0,32 \pm 0,"12$	FRANZ ⁶⁸
	$-0,11 \pm 0,14$	»
	0,0	BARNARD ⁷²
	$+0,171 \pm 0,051$	BOHLIN ²³
	$+0,132$	»
M 31	0,0	BARNARD ¹⁷
	$0,004 \pm 0,005$	VAN MAANEN ¹²⁴
	$0,073 \pm 0,055$	STRÖMBERG ²²⁰
M 51	$0,090 \pm 0,060$	»
	$0'',005 \pm 0'',008$	VAN MAANEN ¹²⁰

The first three values of the parallax for M 32 are relative parallaxes for the nova that appeared in the said nebula in 1885. The measurements are uncertain and do not extend over more than two and six months resp., and no great importance will therefore be ascribed to them. The different parallax-measurements for M 32 show so little mutual concordance, that forming a mean out of all the values is perhaps not to be justified. However, a mean value of all the determinations would give as a final result a very small parallax, as consideration must be taken also to negative parallax values, as pointed out by KAPTEYN¹⁰⁴. Especially puzzling is BOHLIN's great positive value for the parallax of the Andromeda nebula, but as VAN MAANEN has not been able to confirm this result, and the great focal-length of the Mount Wilson reflector and the appropriateness of this instrument for parallax-measures show that great importance is to be assigned to VAN MAANEN's parallax-value, it seems not to be improbable, that BOHLIN's parallax must be due to some other phenomena with a yearly period than the parallactic displacement of the object. As STRÖMBERG's measurements of the parallax for M 31 has been made with the use of the same plates and probably also the same method of reduction, as BOHLIN has used, the same remark will also refer to the result found by him.

Of the above mentioned attempts to measure nebular parallaxes the conclusion will be drawn, that the parallax of spiral nebulæ is smaller than $0''.005$. The values in the table also show that the parallaxes come forth smaller, the greater the instruments used at the determination.

2. *Star Counts and Clusters, constituted by Nebulæ.*

P. Götz, in his great investigation, *Untersuchungen über den Andromedanebel*⁷¹, has measured positions for 1259 stars situated within and near the nebula. Besides he has determined the position of a number of objects (knots, condensations, holes, nebulous rays etc.) within the nebula. He has also, on a photographic plate for a region all around the nebula, situated between α $0^h 23^m - 0^h 48^m$ and $\delta + 38^\circ 11' - + 42^\circ 41'$, counted the number of stars, whereat the part of the plate used at the counts has been divided up into 32×32 squares with a side of $8'.4$.

Götz's Tables and Charts show⁷¹ that the distribution of stars in the neighbourhood of the Andromeda nebula must be considered as purely accidental and not showing a systematic arrangement of the kind that has been observed about the Orion-nebula and the nebulæ about ξ Persei, π Cygni, and 12 Monocerotis, and many others¹⁰⁸. On a more detailed examination of the star distribution within the Andromeda nebula, G. finds, on the contrary, a connection indicated between the nebula and the stars situated within its boundaries. Already at the counting of the whole plate the star density proved to be increasing towards the central parts of the nebula, but on account of the strong nebulosity on the plate the faintest stars in these regions were lost. Hence the star numbers in Götz's Table II near the centre are a little too small, while in this table the star density on an average is 32 per square with a side of $8'.4$. Götz, in examining the star-distribution within the nebula found a mean density of 38 expressed in the same surface unit. As, I suppose, we must presume that the nebulous matter will, at any rate in the more central parts of the nebula, exercise an absorbing effect upon stars situated beyond the spiral, G's result seems to speak against the supposition, that the nebula should be *nearer* than the faintest stars counted on the plate on an average, for in that case a decrease of star density towards the centre of the nebula would be expected, on account of the extinction of the faintest stars at the passage of the star light through the nebulous matter. It is true, we may imagine that the occurrence of a *decrease* of the star density as due to this absorption could be compensated by an *increase* of star-density within the nebula, and as no numerical suppositions regarding the value of the absorption coefficient within the nebula will be possible we cannot yet from Götz's counts draw any definitive positive conclusions about the relation of the nebula to the stellar system. A survey of G's chart regarding the distribution of the stars about and in the nebula (loc. cit. Tafel II) seems, however, to speak against the hypothesis, that the nebula should be nearer to us than the faintest of the counted stars on an average seem to be. Under the supposition that the latter is not the case, we shall derive an upper limit for its parallax.

At the provisional determination of the magnitudes of the measured stars GÖTZ has employed SCHILLER's¹⁷⁸ photometric scale. According to this scale he obtains at his measurement a number of 1198 stars brighter than the 16th magnitude. In counting plate B 265, he obtained within the same area 1050 stars, from which follows, that the counted plate includes stars up to $15^m_{.50}$. Out of HERTZSPRUNG's determinations of the photographic magnitudes of the central Pleiades⁸⁶, which closely agree with CHAPMAN and MELOTTE's scale⁸⁶, based upon the Harvard Polar Sequence, we obtain from the 22 stars between $16^m_{.01}$ — $16^m_{.86}$, also measured by DUGAN⁵⁶, the following systematic difference:

$$m_{\text{HERTZSPR.}} - m_{\text{DUGAN}} = + 1^m_{.89}.$$

As, further, in DUGAN's scale, $14^m_{.48}$ corresponds to the magnitude $15^m_{.50}$ in SCHILLER's, we finally obtain the result that GÖTZ's counts include stars brighter than the magnitude 16,37 in the polar sequence scale.

If the number of stars per 49 squares [= $1^\circ \times 1^\circ$] in GÖTZ's Table are summed up for the part of the plate situated between $\alpha 0^h 43^m_{.8}$ — $0^h 26^m_{.4}$ and $\delta + 39^\circ 0'$ — $+ 42^\circ 1'$, we obtain the following numbers:

1474	1466	1498
1642	1571	1714
1614	1514	1579

or, on an average, 1563 stars pr square degree. Using VAN RHIJN's table¹⁶⁴, we obtain, when the Andromeda nebula has a galactic latitude of -21° , a limiting magnitude for the stars on GÖTZ's plate equivalent to $16^m_{.56}$, consequently in good accordance with the determination performed above.

If the supposition here made, that the nebula is farther away than the faintest stars on GÖTZ's plate, is correct, the parallax of the nebula should be smaller than the mean parallax for stars of the magnitude 16,5. There exists no determination of this mean parallax, but if we extrapolate from KAPTEYN's most recent determination of mean parallaxes¹⁷⁹, we obtain:

$$\pi_{16,5} = 0'',000360.$$

Measuring the positions of 517 small nebulae in the neighbourhood of M 33, WOLF was struck by their peculiar distribution in relation to the nebula. He found that the measured nebulae were grouped in chains which seemed to constitute an extension of M 33's spiral arms. In order to throw some light upon this remarkable discovery, I have with the aid of WOLF's catalogue drawn up a map representing the distribution on the plate of the examined nebulae. We there find how the nebulae seem to emanate from the spiral arms and that between them channels free from every nebula might be drawn on to the central nebula. It is of course not known, which class of nebulae the measured objects belong to, but from their oblong form (WOLF's

g- and *h*-types) we may infer that they are probably of a spiral form, and that, consequently, M 33 is the nucleus of a primary spiral form which consists of secondary spirals. Nor is it quite implausible that some of these small nebulae are star clusters and planetaries,¹ and that we consequently in M 33 and its surrounding nebulae have to deal with a formation analogous to our Galaxy. It also deserves to be emphasized, that the number of nebulae per unit of area is considerably greater on one half of the map than on the other, and that we here might possibly have to see an analogy to the asymmetrical distribution of globular clusters with respect to the stellar system.

Table VIII.

Distribution of stars in and near Messier 33 according to counts on WOLF-PALISA Chart No. 62.

The cross designates the centrum of Messier 33:

$$\alpha = 1^h 30^m,0 \text{ (1875,0)}$$

$$\delta = + 30^\circ 0'.$$

One square has a side of 8'.2.

9	9	7	9	10	5	11	13	12	9	7	14	7	13	15	8	16	7	14	9	9	12	11	6	13	10
5	8	8	10	6	10	14	15	9	16	13	16	7	12	9	16	15	15	12	14	9	13	10	7	8	9
11	12	8	10	17	12	12	12	9	11	12	10	13	11	11	8	9	8	14	16	8	5	14	8	14	7
6	12	9	9	15	21	14	10	9	6	16	17	10	6	11	10	7	12	9	9	9	14	4	6	7	8
11	11	13	17	8	16	16	11	10	10	13	10	12	9	12	10	9	13	10	12	14	8	7	14	9	10
14	12	12	8	15	16	8	10	12	14	16	16	9	13	10	17	11	14	7	9	10	4	6	7	7	15
14	17	9	11	14	11	12	15	8	15	11	12	8	14	12	11	8	6	7	7	8	15	12	6	10	11
10	22	8	10	16	17	10	14	14	8	14	8	8	7	13	8	9	7	17	8	6	7	5	16	16	7
8	14	17	15	9	15	11	13	13	7	12	12	18	14	12	17	14	12	12	17	10	11	10	11	19	8
16	9	7	14	12	13	4	14	7	14	11	14	11	6	10	9	15	10	10	10	11	9	15	17	13	16
7	11	8	12	9	15	7	20	7	18	11	9	8	15	8	11	20	17	9	12	6	12	12	5	9	7
16	14	15	9	15	15	12	14	5	12	19	10	10	11	12	10	13	12	8	7	11	19	10	15	12	6
15	13	13	10	13	9	7	11	14	17	10	7	9	7	9	5	11	10	15	7	11	10	7	7	7	12
12	16	15	10	6	5	9	6	13	13	9	15	4	13	6	12	7	12	6	8	9	7	9	12	9	5
11	16	13	13	10	17	17	11	10	6	9	11	9 ⁺	8	13	9	8	16	7	6	11	7	12	10	11	15
14	11	7	18	14	16	10	10	9	8	7	9	11	9	7	13	11	13	9	13	9	8	17	13	9	7
13	12	9	7	15	12	11	11	3	7	13	7	10	10	12	12	6	8	9	12	11	13	15	11	9	10
8	15	11	12	19	8	14	10	7	8	8	10	6	9	11	11	10	9	11	10	12	7	9	9	11	9
16	12	11	7	9	10	11	11	8	9	4	5	8	7	8	11	14	7	12	10	9	13	11	7	14	7
12	10	15	12	7	19	10	14	13	15	7	12	10	9	7	13	7	12	12	7	10	12	11	5	12	5
9	11	17	19	16	11	11	12	7	11	10	8	10	7	10	9	13	11	8	8	10	9	9	7	14	7
13	6	13	9	15	12	8	13	12	13	12	12	8	6	9	14	15	12	5	9	9	5	12	12	8	11
15	13	9	10	12	11	10	13	14	16	11	6	13	8	9	12	14	11	6	11	13	8	9	7	13	9
11	13	12	12	13	7	8	12	13	10	10	8	11	9	9	8	10	9	10	10	11	12	9	14	7	13
13	9	20	11	24	12	11	10	14	12	7	10	7	3	10	12	12	7	12	9	14	7	17	6	7	12
15	12	14	10	14	7	10	12	11	13	14	14	14	6	18	18	11	13	11	14	9	13	12	7	14	11

¹ N. G. C. 604, situated in one of the spiral arms of M 33, is known as a planetary nebula.

Table IX.

Distribution of stars in and near Messier 33 as resulting from counts on an Upsala Astrographic plate.

The cross designates the centre of the plate and has the coordinates:

$$\alpha = 1^h 25^m.5 \text{ (1875}_0\text{)}$$

$$\delta = +29^\circ 58'.$$

One square has a side of $9'.0$.

15	8	8	13	4	5	7	10	3	11	5	7	5	10					[7]
13	6	10	6	7	16	13	11	10	9	13	9	9	8	12	13	10	8	10
16	5	9	8	9	4	12	11	9	12	9	10	9	12	12	13	9	10	9
10	6	7	9	7	7	10	13	13	12	7	6	11	5	11	12	12	10	10
13	9	6	10	8	10	9	7	10	11	7	11	10	7	6	10	3	14	13
10	11	10	5	9	12	11	7	6	13	10	9	9	12	9	9	8	11	11
9	7	8	10	8	10	11	12	7	6	12	13	7	8	11	14	9	12	15
6	14	7	10	8	7	7	11	14	13	10	4	13	8	13	14	11	10	8
9	10	15	10	13	7	12	12	12	12	6	12	10	11	12	12	14	10	6
11	11	7	9	6	6	12	10	7	6	10	9	8	6	11	5	15	11	9
8	15	8	12	11	10	7	10	8	11	7	11	13	9	11	10	10	12	9
7	7	6	9	9	11	9	10	9	8	12	7	5	12	14	11	12	10	11
9	10	8	9	13	7	10	14	6	10	11	9	8	15	9	13	13	17	17
8	13	8	13	9	8	13	13	13	13	13	10	10	15	9	10	14	11	8
9	10	10	14	7	6	11	10	9 ⁺	8	10	11	11	10	11	8	14	10	10
10	11	8	7	10	13	11	17	10	11	14	10	11	11	8	10	12	5	11
9	7	6	10	14	8	11	14	9	10	10	13	6	8	12	9	14	13	12
8	11	10	5	12	8	13	12	5	8	10	10	13	10	8	7	10	15	8
9	10	9	9	10	10	15	12	10	8	10	10	11	9	11	12	12	16	9
8	8	17	9	11	13	12	11	11	9	11	9	12	11	13	4	10	12	7
10	6	8	14	10	8	12	18	9	9	11	10	11	12	10	14	15	8	6
9	6	7	9	10	5	9	13	10	15	10	12	12	9	22	16	10	6	9
12	11	10	9	9	11	16	11	11	11	9	15	12	11	11	17	8	6	10
8	9	8	8	10	8	9	17	12	8	11	14	11	12	10	9	9	12	5
9	6	9	11	13	10	14	12	11	9	12	8	3	13	15	13	9	13	7
11	13	9	10	11	6	10	13	12	14	12	14	12	13	15	10	10	15	11
7	8	7	13	10	9	14	12	10	14	14	13	9	7	11	9	15	12	8
6	10	11	17	8	6	12	14	12	15	11	11	9	9	7	11	9	11	9
[10]	[13]	[10]	[12]	13	10	16	16	11	10	13	10	7	[7]	[11]	[8]	[5]	[9]	[8]

It is easy to imagine the origin of the above-mentioned channels free from nebulae about M 33 as caused by influence from absorbing media about the nebula. On one of my plates taken with the twin 6-inch astrographic telescope at the Upsala Observatory exposed for 5 hours, I have therefore counted the stars falling within squares with a side of 4 mm. ($9'.0$). (Table IX).

A similar count has been performed on the WOLF-PALISA chart No. 62, Table VIII. Owing to the difficulties in the latter case to distinguish between small nebulae and stars, I have not wanted to assign any great importance to these counts.

The counts on the Upsala plate are performed twice with an interval of some time and are certainly not so reliable as the star counts performed here concerning the star cloud in Cygnus, at which the longest exposures have been double in order easily to distinguish the faintest stars on the plates, but they will nevertheless be worth some confidence. As the ratio between aperture and focal length for the instrument employed at Upsala is $\frac{1}{10}$, the distinguishing at the countings between stars and small nebulae come off rather successfully, and the numbers given in Table IX might therefore pretty well represent the real distribution of the stars.

It appears from the said Table that the stars seem to be distributed at random and that the peculiar distribution of the nebulae is not to be recognized. Nor can

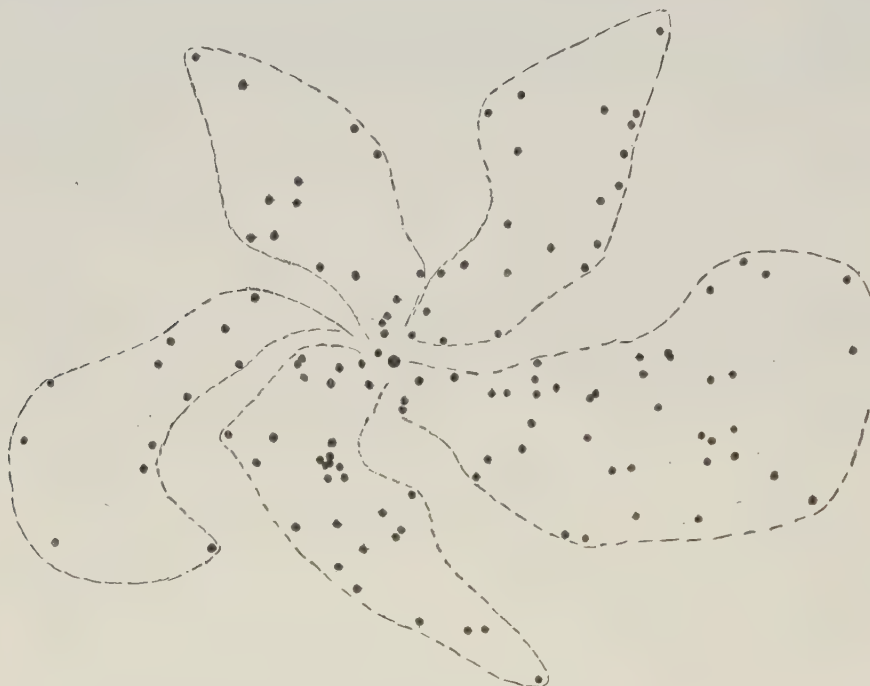


Fig. 5. The Cluster of nebulae in Perseus (coordinates for centre: $\alpha = 3^h 10^m 0^s$; $\delta = +40^\circ 56'$).

a decrease of the star density towards the centre of the nebula be proved and thus it will also in this case be justified to suppose that M 33 is more distant than the faintest stars on our plate.

From Table IX we find that the star density per square degree is 506. Since the galactic latitude of the nebula is -31° , we obtain from VAN RHIJN's table the limiting magnitude for the stars on the plate $= 15^m.72$. From this follows that if $\pi_{15.7} = 0''.000498$, the parallax of the nebula should be smaller than this value.

From the map drawn of the cluster of nebulae in Perseus found by WOLF, it also results that we can distinguish a certain spiral arrangement of nebulae in this cluster; in any case a certain regular distribution is prevailing.

It deserves at last to be mentioned that WOLF has investigated the spectra of some of the nebulae in the large nebulous cluster close to the pole of the Milky Way

and found that they have a continuous spectrum. He has also observed some traces of lines in his spectrograms, but it has not been possible to perform any measures. The form of most of the nebulae examined in that region makes it quite probable that most of the objects are spiral nebulae, and in some cases the spiral structure is quite distinct. It does not seem implausible, as WOLF himself points out²⁴⁴, that the whole cluster of nebulae has a spiral structure. Also in this case an asymmetrical distribution of the nebulae belonging to the cluster is to be traced.

3. *Jeans' Theory.*

JEANS starts from the result found by ROCHE in 1855, that a rotating gaseous mass with high central condensation is flattened out more and more, until it assumes the shape of a lens with a sharp periphery, from which, if the speed increases further, matter will be ejected. If we consider the tidal forces, exercised by distant stars at the rotation of the nebula, JEANS shows how a particle, thrown out from the periphery, instead of describing an elliptic orbit of small excentricity, will, owing to the influence from distant stars, be gradually driven away from the central mass, in describing an orbit that satisfies the equation:

$$r = ae^{\beta\theta} + \epsilon \cos(\theta - \eta), \quad (11)$$

viz. a deformed logarithmic spiral.

Just as LAPLACE'S ring will in course of time become instable and break up into masses, which concentrate into planets, the filaments ejected from the central mass will also, according to JEANS, become instable, if the mass per length-unit amounts to a certain value. In these filaments nuclei will be condensed at tolerably regular intervals and form a chain of detached masses in the spiral arms. We find that such nuclei appear in the arms of most spiral nebulae. In the nebula M 81 RITCHEY thus has counted no less than 2400 nebulous stars.

JEANS now considers M 101, for which VAN MAANEN has found a rotation period of 85000 years. For spiral nebulae he estimates the well-known expression

$$\frac{\omega^2}{2\pi/\rho}$$

to 0.32 and obtains a mean density of $4 \cdot 10^{-17}$.

Further he approximates the density in the spiral arms to about 10^{-17} and then obtains that the nuclei ought to occur at an interval of 0.03 parsec, if they occur at all. In M 101 the nuclei are ill-defined, but JEANS estimates their mean distance to 5'' and thus finds a hypothetical parallax of

$$0'',0007.$$

The forms of spiral nebulae have been examined by VON DER PAHLEN for M 33, 74 and 51¹⁵⁴. He finds that they may be represented by logarithmic spirals. Slight deviations occur, however, and it would perhaps be a desirable thing to examine

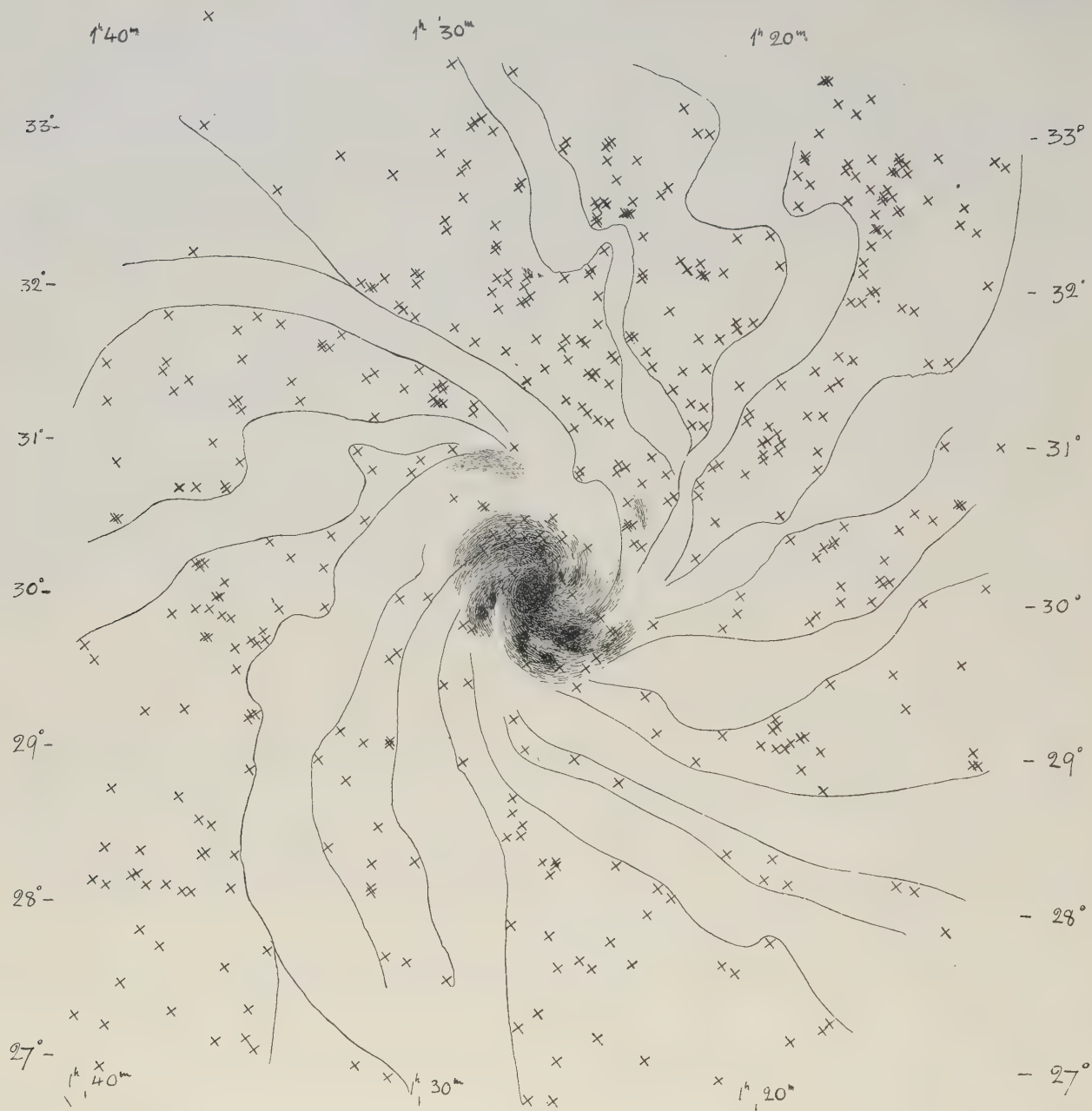


Fig. 6. The nebula Messier 33 and the positions of 517 small nebulae measured by MAX WOLF.

whether a number of well-known spiral nebulae might also be represented by logarithmic spirals deformed with an elliptical term.

In the spirals we should consequently have to see star-producing mechanisms, and as the final state of these a flattened globular cluster would appear. From the present investigation it seems to result, that the spirals are at a greater distance than what corresponds to JEANS' hypothetic parallax for M 101, but probably one may allow a far greater distance for these objects without conflicting with JEANS' theory, which must operate with suppositions not proved; but nevertheless it seems

very plausible, not least because it admits of an explanation of the spiral form and the frequent occurrence of *two* spiral arms without resorting to any other expression for the law of gravitation than NEWTON's.

4. *The spectral and photometric Researches concerning Spiral Nebulæ.*

In 1898, SCHEINER succeeded in obtaining a spectrogram of the Andromeda nebula and thereby he found, in accordance with Sir W. and Lady HUGGINS, that the nebula showed a continuous spectrum with absorption lines of the kind appearing in stars of the *G*-type. SCHEINER called attention to the fact that his observations might be explained by supposing the Andromeda nebula as a galactic system.

Afterwards E. A. FATH⁶³⁻⁶⁵, WOLF²⁴⁸⁻²⁵⁰ and SLIPHER²¹⁰⁻²¹⁴ have with great success devoted themselves to spectral researches of nebulæ and we now know the spectral type for 13 spirals.

During the last few years LUNDMARK and LINDBLAD¹¹⁸ have determined the spectral types for nebulæ by means of the effective wave-length method and obtained the result that the examined spirals generally give $\lambda_{\text{eff.}}$ between 420 and 436 μ , corresponding to the spectral classes *G*—*K*. We will here collect the results hitherto published regarding the spectra and the effective wave-lengths for objects known as spiral nebulæ.

Table X.

N. G. C.	Observed type of spectrum	Observers	$\lambda_{\text{eff.}}$	Spectrum inferred	Colour-index inferred	Magnitude (HOLETSCHEK)
221			432 μ	<i>G</i> ₈	$\overset{m}{+ 0,91}$	$\overset{m}{8,7}$
224	<i>G</i> — <i>K</i>	WOLF, FATH	430	<i>G</i> ₄	+ 0,74	5,0
650,1	<i>G</i>	FATH	425?	<i>F</i> ₆ ?	+ 0,45?	9,4
1023	<i>G</i>	»				9,7
1068	<i>G</i>	»	420	<i>F</i> ₀	+ 0,28	8,7
2841			433	<i>K</i> ₀	+ 1,00	9,1
2903			424	<i>F</i> ₅	+ 0,42	9,0
3031	<i>K</i>	FATH	433	<i>K</i> ₀	+ 1,00	8,0
3034			422	<i>F</i> ₂	+ 0,34	8,8
3623			434	<i>K</i> ₁	+ 1,03	8,9
3627			421	<i>F</i> ₁	+ 0,31	8,6
4214	<i>G</i> ?	SLIPHER				9,3
4251						9,5
4449	<i>G</i>	WOLF, SLIPHER				9,0
4594	<i>F</i> ₅ , <i>G</i>	PEASE, FATH				8,7
4725	<i>G</i>	FATH	430	<i>G</i> ₄	+ 0,74	8,7
4736	<i>G</i>	»	432	<i>G</i> ₈	+ 0,91	7,7
4826	<i>G</i>	»	434	<i>K</i> ₁	+ 1,03	8,6
5194	<i>K</i>	»	436	<i>K</i> ₂	+ 1,11	8,4
5457,8			431 μ	<i>G</i> ₆	+ 0,82	9,2
7331	<i>G</i> ?	»				9,3

By measuring $\lambda_{\text{eff.}}$ of nebulæ it has been possible considerably to advance our spectral knowledge of these objects. To obtain a spectrogram admitting a determination of the spectral type has formerly been possible only for a few of the greatest spiral nebulæ with the aid of very powerful instruments and especially long exposures, while with our method spectral types and colour-indices will be attainable even for fainter objects with rather small instruments and relatively short exposures. The 44 objects, nebulæ and clusters, examined by us, have been measured on plates taken with the Upsala 6-inch twin telescope ($f=150$ cm.) with a total exposure of 157 hours. As, however, the same object is sometimes occurring on more than one plate and with several exposures, the average exposure-time for the objects that can be reached with our instrument need not be put higher than 3 hours. Our catalogue of objects, the $\lambda_{\text{eff.}}$ of which ought to be attainable with the instrument, holds about 500 objects, most of them certainly spirals.

The objects yet measured show that the effective wave-length of planetary nebulæ is $<416^{\mu\mu}$ and as an average, $\lambda_{\text{eff.}}=413^{\mu\mu}$. The spirals give wave-lengths between $420^{\mu\mu}$ and $436^{\mu\mu}$, and in the average, $\lambda_{\text{eff.}}=429^{\mu\mu}$ (spectral type G_2). This gives a method of determining whether a small nebula without a distinct structure belongs to the planetary or spiral nebulæ, and it seems not improbable that the method will also prove useful for distinguishing different types among the spiral nebulæ.

LUNDMARK and LINDBLAD have thus measured 12 nebulæ, whose nebular type is unknown but which certainly are to be included among the spirals on account of the high values of their $\lambda_{\text{eff.}}$

Table XI.

N. G. C.	$\lambda_{\text{eff.}}$	Spectrum inferred	Colour-index inferred	Magnitude (HOLETSCHEK)	Description according to N. G. C.
3077	$424^{\mu\mu}$	F_5	$+0,42$	$10,2$	cB, cL, mbM, R with ray
3377	433	K_0	$+1,00$	9,8	$vB, cL, lE, svmbMBN$
3379	432	G_8	$+0,91$	9,1	$vB, cL, R, psbM, r$
3384	428	G_0	$+0,56$	9,5	$vB, L, R, psmbM$, 2nd of 3
3412	427	F_9	$+0,53$	9,5	$B, S, lE 135^\circ \pm, smbMN$
3613	429	K_6	$+1,20$	10,5	$vB, cL, mE 305^\circ, smbMN$
3619	439	K_6	$+1,20$	11,0	$cB, cL, R, vgm bM$
4278	424	F_5	$+0,42$	9,6	vB, pL, R, mbM, r , 1st of 3
4494	428	G_0	$+0,56$	9,2	$vB, pL, R, vsmbMN$
5471	431	G_6	$+0,82$	—	$F, S, R, *12.13p$
5473	429	G_2	$+0,64$	10,7	pB, S, R, gbM
5485	$430^{\mu\mu}$	G_4	$+0,74$	11,0	$cB, R, vgbMf$ of 2

For further details concerning the measures and reduction of $\lambda_{\text{eff.}}$ for nebulæ, see Aph. J. 46,205 1917 and 50 (in press).

The measurements shown in Table X give an interesting contribution to the question of the selective absorption in space. The spectral types having been determined from the appearance of the lines, and, on the contrary, λ_{eff} from the centre of gravity of the continuous spectrum, we may expect that, in case there is a selective absorption prevailing in space, the λ_{eff} s ought to be displaced towards the red end of the spectrum, and, consequently, the inferred spectral-types redder than the observed ones. As the inferred spectra cannot very well be displaced relatively to the observed ones with more than $1/2$ spectral interval (or $0^m_{,2}$ in colour-index) at the utmost, and as the parallax of the tabulated spiral nebulae must be considered $< 0''_{,000005}$, we conclude that the change in colour index per parsec, d , is

$$< 0^m_{,0000001}.$$

SHAPLEY, in a somewhat other way, finds $d < 0^m_{,000002}$.

Thus it is evident that, if there exists a selective absorption in space, it is so small that it is not necessary to consider it, as long as greater distances are not concerned than those here derived for globular clusters and spiral nebulae.

FATH's investigation⁶⁵ concerning the total spectrum of the Milky Way has given the result that our stellar system in its entirety seems to belong to the spectral type G . This circumstance both confirms the above discussion about space absorption and also lends some probability to the supposition that spiral nebulae are remote galaxies.

The discovery lately made by EVERSHED⁶² of the occurrence of stationary calcium clouds in the Milky Way will perhaps throw new light upon the appearance of the H and K lines in the spectra of spiral nebulae. If it should prove to be the case that the occurrence of such clouds is something characteristic of the Milky Way, it is near at hand to see in the appearance of the said lines in the spectra of spirals a support for the opinion that we have here to do with galactic systems or large stellar systems. On the other hand, the fact that in certain spiral nebulae there appear bright lines characteristic of the radiation of the WOLF-RAYET stars and the gaseous nebulae, seems to be an argument against this opinion, as it does not seem probable that the objects, which in our stellar systems give emission-lines in their spectra would have such a total magnitude, that they would come in sight in a total spectrum of the Milky Way examined from a point far outside of the same.

To this, however, the objection might be made, that the emission-lines have not been observed in all the spirals examined, and there is nothing preventing that some of them contain considerably more nebulous matter, planetaries and WOLF-RAYET stars than our Galaxy.

If we do not adopt the opinion that spiral nebulae are stellar systems, there only remains the possibility of a glowing (star-) nucleus, globular in form, and surrounded by a gaseous atmosphere that reflects the light from the nucleus. REYNOLDS¹⁶²⁻¹⁶³, who maintains this opinion, has photographically derived the light-curve for the Andromeda nebula. From his intensities, derived by taking the mean

between points situated symmetrically about the nucleus, we find that the following expression can be derived, which pretty well satisfies the observations with the exception of the two points next to the nucleus.

$$m = 5 \log h + 1,78. \quad (12)$$

In this formula m designates the apparent magnitude of a point in the nebula and h its distance from the centre in mm. How REYNOLDS' measures can be represented by the formula is shown by the following table.

$5 \log h$	$m_{\text{obs.}} - m_{\text{calc.}}$	$5 \log h$	$m_{\text{obs.}} - m_{\text{calc.}}$
	m		m
3,89	+ 0,23	2,38	+ 0,04
3,70	+ 0,04	1,98	+ 0,07
3,49	+ 0,10	1,50	- 0,13
3,27	+ 0,06	0,88	- 0,08
3,01	- 0,09	0,00	- 0,41
2,72	+ 0,01	-1,51	-

This law is the same as the one found by BERGSTRAND²¹ for the light-curve of the solar corona, and implies that the surface-intensity decreases inversely with the square of the distance from the nucleus.

If we do not take into consideration that at different distances from the nucleus the nebula has certainly not the same extent in a direction at right angles with the plane of the spiral arms, we may as an approximation consider the intensities obtained by REYNOLDS as a measure for the variation of the star density in the supposed Andromeda-galaxy from the nucleus and outwards. We suppose the true dimensions of the nebula to be 28000 light-years in accordance with the parallax-determinations from the 11 temporary stars that have appeared in it, and then obtain, from REYNOLDS' measurements and SCHWARZSCHILD's treatise¹⁸³ concerning the variation of the star density in our stellar system, the two curves in fig. 7.

Owing to the great uncertainty regarding our knowledge of the decrease of star density within our stellar system, as well as the uncertainty of the value we have employed for the true dimensions of the nebula, no certain conclusions are to be drawn from the two curves. Yet, it seems to be evident that the supposed Andromeda-galaxy has a central condensation, to which our stellar system has no analogy. Probably the distribution of the light (star density respectively) within M 33 is rather to be compared with that of our Galaxy.

A consequence of REYNOLDS' theory is, that the reflected light from the gaseous atmosphere of the nebulae ought to be polarized. He has also made observations, both visually and photographically, in order to examine this matter and he considers, regarding M 64, that he has found some indices of polarization. Owing to the many difficulties involved in these experiments, it might be considered that

REYNOLDS' observations, yet in themselves very valuable, do not in this case form any definitive proof against the cluster-theory.

The difference in colour, resp. spectral type, found by SEARES and LUNDMARK & LINDBLAD, between the spiral arms and the nucleus in some spiral nebulae, will perhaps be regarded as speaking against REYNOLDS' opinion, as a reflection on the particles of the nebulous atmosphere would result in making the atmosphere redder than the nucleus, while the observations above-mentioned show, that it is just the contrary that is the case.

Are the spectral and photometrical researches concerning the spiral nebulae not able conclusively to confirm the theory of their great distances, we must nevertheless

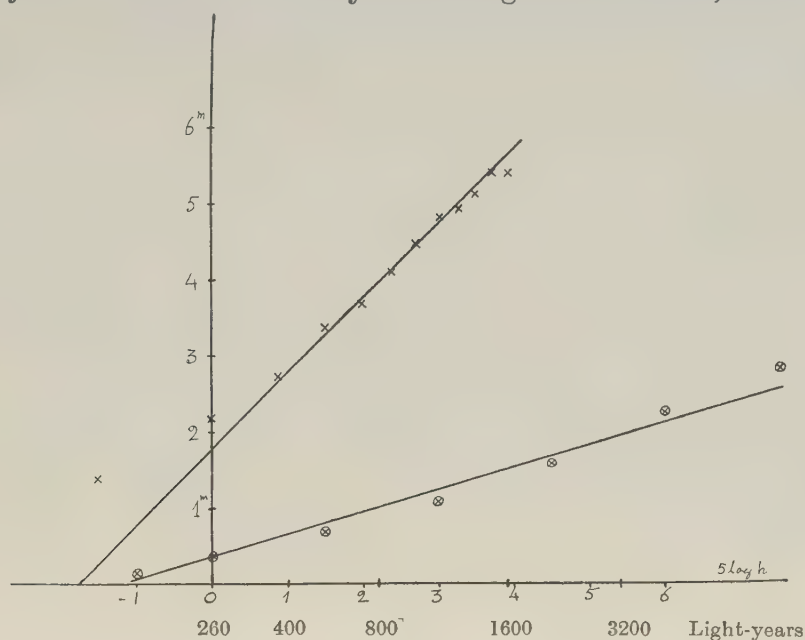


Fig. 7.

Upper curve: The light curve of Andromeda nebula.

Lower curve: The star-density curve in our stellar system derived from SCHWARZSCHILD's treatise A. N. 190:361.

say that, if REYNOLDS' not very plausible theory is correct, nothing else is to be supposed than that the spirals must be at distances at least equal to those of the globular clusters. If not, implausible suppositions will have to be made concerning the physical state of the stars in the spiral nebulae.

5. *The Motions of Spiral Nebulae.*

It is obviously by no means out of the question, that the great radial velocities found for the spirals are not real, but that the measured displacements of spectral lines represent some other phenomenon than a DOPPLER-effect. We want, however, to call in mind that some stars have a radial velocity amounting to the same order of

magnitude as those of the nebulæ, and that, on an average, the radial motions of nebulæ are only 9 times as great as the velocities that it has been possible to examine independently of spectroscopical methods (e. g. rotations of planets, radial velocities of planets and comets).

Unfortunately, there exists no older extensive series of nebular positions than the determination of the coordinates of 500 nebulæ performed in 1863—74 by SCHULTZ¹⁸¹ at Upsala. These measurements have been reduced anew by WIRTZ²²⁷, who in extensive investigations has also discussed later measurements and derived proper motions for the nebulæ and calculated the apex-value mentioned above.

For the 98 small nebulæ, probably spirals, investigated by WIRTZ, we obtain the following value for the mean proper motion:

$$\mu_m = 0'',036$$

CURTIS^{46, 47}, who has made extensive investigations of the proper motions of spiral nebulæ by measuring CROSSLEY plates with an average interval of 13,85 years, has published a communication regarding his results and finds, for 66 great spirals:

$$\mu_m = 0'',033.$$

A few other results are further to be found concerning the lateral displacements of spiral nebulæ. BARNARD¹⁷ in A. J. 715 has communicated the results of his micrometrical measurements regarding the Andromeda nebula, and finds that during the 20 years covered by his measurements no proper motion can be shown. He gives the following positions for the nebula, referred to one of his comparison stars:

Epoch	Distance	Angle of Position
1836	125'',5	261°,11
1867	124,75	261,28
1898	125,42	261,28
1898,7	124,67	261,28
1909,9	124,67	261,31
1916,3	124,63	261,29

From this measures results $\mu \leq 0'',004$.

LAMPLAND¹¹² has measured a series of photographs of the Andromeda nebula, and obtains for the proper motion of the nucleus $\mu = 0'',019$. BOHLIN in his parallax-determination obtained $\mu_\alpha = 0'',090 \pm 0,080$, while VAN MAANEN obtained $\mu_\alpha = -0'',0013 \pm 0'',0011$. If we take this value into account, we obtain from the two parallax-determinations as a mean $\mu = 0'',040$. From a discussion of 11 meridian observations distributed over the years 1800—1892, BOHLIN obtains $\mu = 0'',010$. A simple mean from these four determinations has the value:

$$\mu_{\text{Andr.}} = 0'',018.$$

From his measurements of internal motions in M 101 VAN MAANEN¹²¹ derives a common proper motion for the examined points and, consequently, for the nebula as a whole:

$$\mu_{101} = 0'',013.$$

LAMPLAND has found for M 51, on an average, $\mu = 0'',081$. VAN MAANEN at his parallax-determination finds for μ_α , which is, according to LAMPLAND, the greatest velocity component the value $0'',003$.

As a mean we thus obtain:

$$\mu_{51} = 0'',040.$$

For M 99 LAMPLAND obtains:

$$\mu_{99} = 0'',043,$$

a value which he considers uncertain, and to which he does not assign any great weight.

The same author finds for N. G. C. 4594:

$$\mu_{4594} = 0'',088.$$

For N. G. C. 4565 and 7814 he does not find any trace of proper motion.

Summing up, we obtain:

	μ
N. G. C. 224	$0'',018$
5194	$0,040$
4254	$0,043$
5457	$0,013$
4594	$0,088$
4565	$0,00$
7814	$0,00$
	$0'',029$

Excluding the last two, we obtain $\mu = 0'',040$.

It will be unnecessary to mention that the above values must be considered as very uncertain, and that the authors themselves have made a reservation against attaching too great importance to them.

If we write down the formula (2):

$$\pi_m = 4,74 \frac{\mu_m}{V_t}, \quad (13)$$

where $V_t = \frac{\pi}{2} V_m^{32}$, we obtain, as V_m according to Table IV is 490 km./sec., the following three determinations of π_m :

μ_m	π_m	
0'',036	0'',000356	(from WIRTZ' 98 small nebulæ)
0,033	319	(from CURTIS' 66 spiral nebulæ)
0,029	0,000178	(from the proper motions of 7 spiral nebulæ).

Certainly these proper motions, and, consequently, also π_m , are too great.

Another way of determining π_m is obtained by comparing the speed of the solar motion V_0 in relation to the system of spiral nebulæ resulting from our Apex-determinations with the value of the same quantity obtained by WIRTZ from proper motions of the 98 small (spiral) nebulæ mentioned above.

According to this we find:

$$\pi_m = 0'',000163.$$

6. *Measurements of Rotations and Internal Motions of the Spiral Nebulæ.*

Several attempts were made during the 18th century to determine internal motion in spiral nebulæ but always gave negative results. With the development of spectral analysis it became possible to detect the rotation of spiral nebulæ a few years ago. In 1914 thus SLIPHER²¹⁰ discovered rotations in the spindle nebula N. G. C. 4594, and at the same time WOLF²⁵⁰ found in M 81 a radial velocity component due to rotation. Shortly afterwards PEASE¹³⁹ communicated the discovery of internal motions in M 33.

By measuring in the stereocomparator four pairs of plates taken at the Lick and Mount Wilson Observatories during the interval 1899—1915, VAN MAANEN¹²¹ has succeeded in establishing an annual rotational component of 0'',022 in M 101 at a mean distance of 300'' from the centre, from which follows a rotation period of 85000 years for a particle situated at the said distance from the nucleus.

KOSTINSKY¹⁰⁷ has communicated that he has by means of the same method found internal motion in M 51 and that the motions amount on an average to 0'',04—0'',05 yearly. LAMPLAND has also tried to derive internal motion in the same nebula, and considers that he has found from his measures, which he, however, admits to be very uncertain, that the secondary nucleus (N. G. C. 5195) of this nebula performs a revolution about the nucleus in a period of 43000—108000 years.

Supposing that the above components of rotation are on an average of the same order of magnitude as the components of rotation found by measuring the radial velocities, we obtain a means of estimating the average parallax of the nebulæ. The following results regarding the component of rotation in the direction of the vision-radius have hitherto been published.

Object	Rotational component	Distance from the nucleus	Observer
M 81	100 km./sec.	»Near«	WOLF
N. G. C. 4594	330	120"	PEASE
»	(600)	120	SLIPHER
M 31	88	121	PEASE
Mean	173	120	

KOSTINSKY has not stated at which distance from the nucleus the rotation for M 51, found by him, is valid, but we may suppose, that it is for the distance of 300". We obtain from LAMPLAND's data $\mu_{\text{rot}} = 0''.025$, and as a mean between the values of VAN MAANEN, KOSTINSKY and LAMPLAND, $\mu_{\text{rot}} = 0''.031$. According to the Table, $V_{\text{rot}} = 173$ km./sec., from which follows

and $\pi_m = 0''.000358$
 $\pi_m = 0''.000221$ (employing SLIPHER's value for N. G. C. 4594).

The measured μ_{rot} are probably too great. Thus CURTIS⁴⁶, in examining the proper motions of 66 large spirals has not been able to find any internal motion in them. A decrease of μ_{rot} would diminish the mean parallax, and our values for the same may certainly be considered as upper limits.

From KEPLER's third law VAN MAANEN derives the following relation between the parallax and a central mass which would cause a rotation of the magnitude he has found for M 101:

$$M = 0.0037 \pi^{-3}.$$

With the adoption of our mean parallax $0''.000221$ we find a central mass of $0.6 \cdot 10^9 \odot$, where $\odot$ designates the mass of the sun. If we call to mind that CHAPMAN and MELOTTE have estimated the number of stars in our stellar system to $2 \cdot 10^9$, and if we consider, that according to RUSSELL's data¹⁷² the stars ought to have on an average a mass not quite as great as that of the sun, we find that the results regarding the masses seem to confirm the theory of the spirals as being remote galactic systems. Their distances however turn out relatively small, but even a slight variation in μ would considerably change the value of π_m , which of course can only be considered as a rough estimate.

7. Wolf's Measurements of the Clouds in the Milky Way and in the Spiral Nebulae.

Supposing that the spiral nebulae are remote galactic systems of a structure comparable with that of the Milky Way, MAX WOLF²⁴⁷ has, with every reservation, given a way of deducing parallaxes for some of the large spiral nebulae. He assumes that the so-called dark nebulae in the Milky Way (for instance the clouds near and

in the Orion nebula) are formations of a kind that occurs also in the spiral nebulae, and that in both cases the absolute dimensions are on an average the same. By measuring the breadth of the »Längshöhlen» and »Querhöhlen» (length- and cross-caves, or rifts) he derives relative parallaxes for 8 of the large spiral nebulae. Then, in order to obtain the true distances, he starts from the distance estimated by CAMPBELL to the Helium stars related to the Great Orion nebula ($\pi = 0''.008$), and concludes, as the breadth of the »Längshöhlen» in the Orion regions is $72'$ and the breadth of the »Querhöhlen» $20'$, that the Andromeda nebula, for which the same values have been estimated to $70''$ and $18''$ respectively, is at a distance of 10000 parsec. By assuming Nova Persei 1901 to be situated in the galactic drift in Perseus, and estimating the average breadth of the »Längshöhlen» to $120'$ and of the »Querhöhlen» to $30'$, and by supposing that the parallax for Nova Persei can be fixed to $0''.01$, he obtains a new determination of the parallaxes of the nearest spiral nebulae that is in close accordance with the first determination. Below WOLF's results (re-calculated) are summed up:

Table XII.

Object	$\pi_{\text{Orion}} = 0''.0071$		$\pi_{\text{Nov. Pers.}} = 0''.010$		Mean
	From the length-caves	From the cross-caves	From the length-caves	From the cross-caves	
N. G. C. 224	$0''.000105$	$0''.000099$	$0''.000096$	$0''.000099$	$0''.000100$
598	38	33	35	33	35
3031	19	19	18	19	19
5457	12	11	11	11	11
5194	11	8	10	7	9
4565	8	—	7	—	7
6946	8	5	7	5	6
2903	$0''.000006$	$0''.000005$	$0''.000005$	$0''.000005$	$0''.000005$

Concerning the parallax-value employed by WOLF for the Orion nebula, it seems to have been confirmed by more recent determinations. Thus KAPTEYN finds, for the Helium stars connected with the Orion nebula, a mean parallax amounting to $0''.0054$, and BERGSTRAND has lately²⁰ determined the mean parallax for a group of 15 bright Helium stars in and near the nebula to $0''.008$. W. H. PICKERING¹⁵⁵ finds a considerably diverging value, $\pi = 0''.0005$. An adoption of this parallax would give a much greater distance for the spiral nebulae than that obtained by WOLF. However, in view of the uncertainty of the data employed by PICKERING, no reasons seem to exist for giving up the value resulting from CAMPBELL's, KAPTEYN's and BERGSTRAND's determinations, which are in good mutual concordance. The hypothesis of the spiral form of the Milky Way, assumed among others by WOLF, is of course rather uncertain, and much speaks in favour of the supposition that the star

clouds in the Milky Way form a unit of far greater dimensions than those derived for the spiral nebulae, with aid of the parallaxes determined by WOLF. Moreover, WOLF remarks that the diameters of the examined spiral nebulae, calculated to about 1000 light-years, turn out surprisingly small. On the other hand, the values of the parallaxes for the Orion nebula and Nova Persei are possibly somewhat too great. As to the latter, we find $\pi = 0''.010$ out of the five determinations used in the following discussion of the appearance of Novæ in the spiral nebulae. As it has not been possible to take into consideration two determinations leaving negative values (AITKEN and HARTWIG), it is probable that the parallax is in reality smaller than the value employed here.

8. *The Appearance of Novæ in the Spiral Nebulae.*

In 1885 HARTWIG, among others, found a nova of the seventh magnitude close to the centre of the Andromeda nebula, and in 1895 Mrs FLEMING communicated the discovery of a nova which had appeared near the nucleus of the spiral nebula N. G. C. 5243. Since 1916 American astronomers, in examining long-exposure plates of spiral

Table XIII.

	Object	Date of observed maximum	Time of discovery	$m_{\max.}$	Distance from nucleus	Discoverer	Authority
1	N. G. C. 224	Aug. 1885	1885	^m 7,2	16''	HARTWIG others	A. N. 112:245
2	5253	July 1895	1895	7,2	32	Mrs FLEMING	Harv. Circ. No. 4
3	4321	Mar. 1901	1917	13,5	110	CURTIS	L. O. B. No. 300
4	3147	Apr. 1904	—	13—14	340	Mrs ISAAC ROBERTS	P. A. S. P. 29:214
5	224	Sept. 1909	1917	16,3	249	RITCHEY	P. A. S. P. 29:211—12
6	224	Sept. 1909	1917	17,0	198	»	P. A. S. P. 29:211—12
7	2841	Feb. 1912	1917	16	54	PEASE	P. A. S. P. 29:213
8	4321	Mar. 1914	1917	14	111	CURTIS	L. O. B. No. 300
9	4527	Mar. 1915	1917	14	45	»	L. O. B. No. 300
10	6946	July 1917	1917	14,6	111	RITCHEY	P. A. S. P. 29:213; Sirius 1918
11	224	Sept. 1917	1917	17,5	600	SHAPLEY	P. A. S. P. 29:213
12	224	Oct. 1917	1917	18,0	227	RITCHEY	P. A. S. P. 30:162—63
13	224	Nov. 1917	1917	16,8	321	»	P. A. S. P. 30:162—63
14	224	Jan. 1918	1918	17,1	143	»	P. A. S. P. 30:162—63
15	224	Feb. 1918	1918	17,7	123	»	P. A. S. P. 30:162—63
16	224	Feb. 1918	1918	17,2	550	DUNCAN	P. A. S. P. 30:255
17	224	Oct. 1918	1918	17,3	466	SANFORD	P. A. S. P. 30:341
18	224	Oct. 1918	1918	17,6	380	»	»
S u s p e c t e d O b j e c t s							
	224	1664	—	6,0?	—	BULLIALDUS	See Bibliography No. 29
	3031	Feb. 1910	1917	18?	—	RITCHEY	P. A. S. P. 29:210—12
	2403	Feb. 1910	1917	16,5?	—	»	»
	5457—8	May 1915	1917	18,5?	—	»	»
	5457—8	May 1915	1917	19?	550	»	»

nebulæ, have found a great number of faint novæ in them, and at present there are no less than 18 objects known, which are evidently to be considered as novæ, appearing in spiral nebulæ.

There cannot be any doubt regarding the fact that these stars are really situated within the spiral nebulæ, as there is no possibility of assuming that 11 novæ should be seen just in the direction of the Andromeda nebula, whereas no novæ have appeared in its neighbourhood. On the other hand, it can by no means be regarded as a certain fact that the stars tabulated above are really novæ, but the hypothesis is near at hand that we have here to deal with variables of long period. In order to contribute to the solution of this question, I have considered it appropriate to try to derive the light-curves in the cases where sufficient observations are at hand. The best-observed star of all in the table is the Nova Andromedæ 1885. I have laid as a basis for its light-curve the following series of observations.

Observer	Number of observations	Observed	References
BACKHOUSE	7	1885 Sept. 1—Oct. 1	M. N. 48: 108—09
BAKHUYZEN	5	Sept. 1—15	A. N. 112: 323
BAXENDELL J. & J.	27	Sept. 3—Nov. 8	Observatory IX: 94—95
CHARLIER	4	Sept. 2—17	A. N. 112: 389, 391
COPELAND	29	Sept. 1—1886 Febr. 2	M. N. 47: 49
ENGELMANN	21	Sept. 1—Nov. 18	A. N. 112: 323—326; 113: 269—70
ESPIN	11	Sept. 27—1886 March 6	Observatory IX: 156—57
FRANZ	12	Aug. 31—Dec. 11	A. N. 118: 123—24
HAGEN	24	Sept. 3—Sept. 22	Sid. Mess. IV: 286
HALL	10	Sept. 29—1886 Febr. 7	Amer. Journ. of Science 1886: 299
HARTWIG	23	Aug. 19—Oct. 19	A. N. 113: 21, 387
MÜLLER	16	Sept. 2—Oct. 13	A. N. 113: 23
PRITCHARD	17	Sept. 7—Oct. 16	M. N. 46: 18—19
PARKHURST	40	Sept. 1—Nov. 4	Sid. Mess. V: 90
RADCLIFFE	35	Sept. 9—Dec. 10	M. N. 46: 56—57.

No very high degree of accuracy may be expected from these observations, because, as a matter of course, they were rendered considerably more difficult by the bright nebulous background. Considerable systematic differences also appear between the different observers, owing to the use of various scales and instruments. Graphically we have derived systematic corrections for the different series, whereupon they have been reduced to PARKHURST's photometric system whose magnitudes will evidently agree with the Harvard visual scale. I have not assigned different weights to the different series of observations. Nor have I considered it appropriate to give to the measurements performed with the Zöllner or wedge photometer (PRITCHARD and others) parentheses any greater weight than is given to the estimations, considering the uncertainty of the determinations brought in by the existence of the light nebulous background.

After the systematic errors have been applied, the observations show a relatively good concordance, and the light-curve derived from the means in fig. 8 may be considered as a comparatively exact one up to the 1 Dec. 1885. The observations obtained after that time are sporadic and, owing to the low magnitudes of the stars, uncertain, and possibly have not been corrected with accuracy for systematic errors.

From the light-curve we conclude that the Nova Andromedæ must with regard to light-fluctuations be considered as having been a typical nova. The magnitude variations of short periods, which are to be seen in the curve, very much resemble the variations in the light-curves of the Nova Persei, Nova Geminorum, and Nova Aquilæ. Also here there is an indication that the period-length increases towards the end of the period of observation.

For Nova Centauri there are only 19 estimations of magnitude known, but the light-curve drawn from them seems, notwithstanding, to give the progress of the variation of magnitude typical of a nova.

From the estimations given by RITCHEY for the Novæ Andromedæ No. 5 and 6 we gather, in spite of the small number of observations, that also these stars will certainly have to be reckoned to the class of novæ. (Fig. 10). CURTIS, with regard to the three novæ discovered by him on the Crossley reflector plates, has been able to give some data of the light-fluctuations of the three stars, which, it is true, do not suffice for drawing a light-curve, but, in any case, for rendering it most improbable that these objects should be variables of long period.

Even if Nova Andromedæ 1885 and Nova Centauri have, with regard to their spectra proved to differ from the spectra of the new stars flashing up in the Milky Way, yet there was no indication that the former stars were to be classified among the variables of long period.

According to CURTIS the value of the colour-index for RITCHEY's nova in N. G. C. 6946 shows that this star cannot be a variable star of long period, and a spectrogram of the same star, obtained by RITCHEY and PEASE with the Mount Wilson

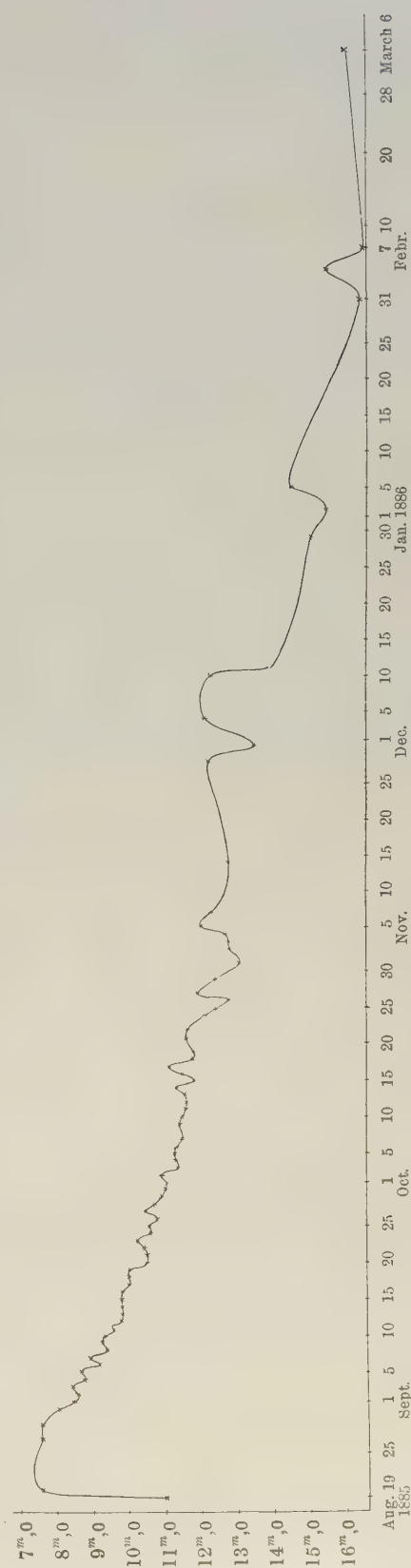


Fig. 8. Curve showing the light-fluctuations in Nova Andromedæ 1885.



Fig. 9. Light-curve for Nova Centauri 1895

reflector, shows a strongly continuous spectrum which is seen crossed by what appears to be a series of bright bands.

As it is from those reasons evident that the new stars flashing up in spiral nebulae are to be regarded as objects of the same kind as the galactic novæ, we will, according to a suggestion by CURTIS^{48, 52}, use them for a determination of the mean parallax of the first seven nebulae of Table XIII, and of the parallax of the Andromeda nebula. If we designate the mean parallax of the spiral nebulae with

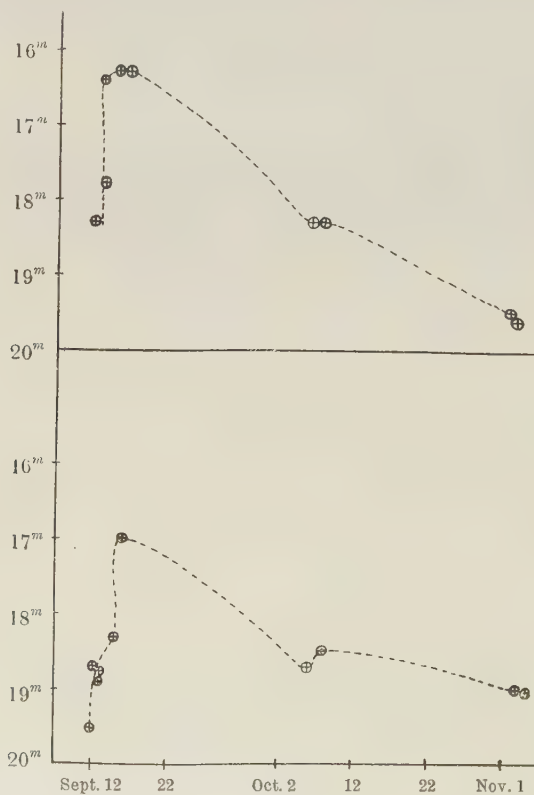


Fig. 10. The light-curves for Nova Andromedæ No. 5 & 6 (RITCHIEY)

$\pi_{sp.}$ and the mean parallax of the novæ in the Milky Way with $\pi_{nov.}$, and M and m signify the absolute and the apparent mean quantisy for the novæ in the Milky Way, and M' and m' the corresponding quantities for the novæ in the spiral nebulæ, we obtain, according to (1):

$$\begin{aligned} M'_{\max.} &= m'_{\max.} + 5 + 5 \log \pi_{sp.}, \\ M_{\max.} &= m_{\max.} + 5 + 5 \log \pi_{nov.} \end{aligned} \quad (14)$$

Supposing that the absolute maximum magnitude of the novæ in the Milky Way is on an average equivalent to the same quantity for the novæ found in the spiral nebulæ, or: $M'_{\max.} = M_{\max.}$, we thus obtain:

$$\log \pi_{sp.} = \log \pi_{nov.} + \frac{1}{5} (m_{\max.} - m'_{\max.}). \quad (15)$$

From the table we find the mean of the maximum magnitude of the 18 novæ = $15^m_{.1}$. As to $\pi_{nov.}$ we will first take into consideration the direct parallax-measurements existing for four galactic novæ.

<i>Nova Persei</i> $\pi_{abs.}$	<i>Nova Lacertæ</i> $\pi_{abs.}$
+ 0'',009 $\pm$ 0'',004 VAN MAANEN ¹²⁰	+ 0'',013 $\pm$ 0'',012 SLOCUM & MITCHELL ²¹⁷
+ 0,028 $\pm$ 0,020 HASSENSTEIN ⁷⁵	— 0,005 $\pm$ 0,020 BALANOWSKY ¹²
— 0,003 $\pm$ 0,023 CHASE ⁴⁰	+ 0'',008 $\pm$ 0'',010
+ 0,036 $\pm$ 0,010 BERGSTRAND ¹⁹	
+ 0,007 $\pm$ 0,004 KÜSTNER ¹⁰⁹	
+ 0'',010 $\pm$ 0'',003	
<i>Nova Geminorum</i> $\pi_{abs.}$	<i>P Cygni</i> $\pi_{abs.}$
+ 0'',012 $\pm$ 0'',007 SLOCUM & MITCHELL ²¹⁷	— 0'',014 $\pm$ 0'',008 SLOCUM & MITCHELL ²¹⁷
— 0,013 $\pm$ 0,013 MILLER ¹²⁹	
+ 0'',006 $\pm$ 0'',006	

These novæ had a maximum magnitude of 0,0, 3,7, 5,0 and 3,0 respectively, or on an average $2^m_{.9}$. We find their mean parallax to be $0'',0068 \pm 0,0025$.

The scanty data existing for the proper motions and radial velocities give, from 8 novæ, $\mu_m = 0'',04$ and $V_m = +20 \text{ km./sec.}$, from which follows $\pi_{nov.} = 0'',0060 \pm 0'',0040$, agreeing closely with the value obtained from the parallax-measurements. As this value corresponds to a mean $m_{\max.}$ of $2^m_{.5}$, we shall assume that for $m_{\max.} = 2^m_{.7}$,

$$\pi_{nov.} = 0'',0066 \pm 0'',0020.$$

We then obtain according to the above formula:

$$\pi_{sp.} = 0'',0000218.$$

P Cygni occupies a peculiar position among the novæ, as its light-variation is much slower than is the case with the typical novæ. Nevertheless, as shown by MERRILL (P. A. S. P. 24: 181), it is really a nova. This is the reason why we have used its parallax for determining $\pi_{\text{nov.}}$ but omitted it at the determination of the difference between the maximum and minimum intensity of the novæ, although by employing it we would not have considerably altered the result.

We consider the following galactic novæ for which measurements exist of their greatest and smallest apparent magnitude:

	$m_{\text{max.}}$	$m_{\text{min.}}$	$m_{\text{min.}} - m_{\text{max.}}$
	m	m	m
T Coronæ	2.0	9.5	7.5
Q Cygni	3	15	12
T Aurigæ	4.3	14	9.7
Nova Sag. 1	4.7	15	10.3
Nova Persei	0.0	13	13
Nova Gem. 1	5.1	16.3	11.2
Nova Lacertæ	5.0	12.5	7.5
Nova Gem. 2	3.7	12	8.3
Nova Aqv. 3	— 1.4	11.5	12.9
	$2^m.9$	$13^m.2$	$10^m.3$

Consequently we obtain the result that $m_{\text{max.}}$ on an average is $2^m.9$, while $m_{\text{min.}}$ on an average is $13^m.2$. If we make the plausible supposition, that the mean magnitude before their appearance is $= m_{\text{min.}}$, and if $M_{\text{min.}}$ is supposed as equivalent to the absolute magnitude of the WOLF-RAYET stars, taken as a mean from the following determinations:

$M_{\text{Wolf-Rayet}} = + 1.2$	VAN MAANEN, from three directly measured parallaxes
— 1.1	KAPTEYN, from stream motion
— 2.8	HERTZSPRUNG, from parallactic motion
— 0.9	

we obtain:

$$\pi_{\text{nov.}} = 0''.00015$$

from which is obtained the following new determination of the mean parallax of spiral nebulæ:

$$\pi_{\text{sp.}} = 0''.0000005.$$

On the presumption, that the novæ employed in this determination have a mean parallax as great as the mean parallax for the O-stars examined by GYLLENBERG⁷⁰, i. e. $\pi_{\text{nov.}} = 0''.0027 \pm 0.00018$, we obtain:

$$\pi_{\text{sp.}} = 0'',000010.$$

If, finally, we suppose that the 9 novæ in the Milky Way included in the table are at an average distance corresponding to the average distance to the stars of the 13th magnitude, we obtain $\pi_{\text{nov.}} = 0'',0011 \pm 0'',0005^{72}$ and:

$$\pi_{\text{sp.}} = 0'',000004.$$

By using all galactic novæ, we would in the average obtain $m_{\text{max.}} = 5^m,7$, which would give smaller parallax-values, if the three last methods were employed. Considering that the novæ flashing up in spiral nebulae must be regarded as belonging to the most brilliant of their kind, I have only employed those galactic novæ for which it has been possible to determine $m_{\text{min.}}$, and especially have considered it necessary to exclude the faint novæ spectroscopically detected.

If, again, we consider the 11 novæ observed in the Andromeda nebula, and if we derive its hypothetical distance under the four suppositions about $\pi_{\text{nov.}}$ we have made previously, we obtain:

$$\begin{array}{r} \pi_{\text{Andr.}} = 0'',0000123 \\ 0,0000003 \\ 0,0000055 \\ 0,0000022 \\ \hline 0'',0000051 \pm 0'',0000018 \text{ (p. e.).} \end{array}$$

A weighted mean would give:

$$\pi_{\text{Andr.}} = 0,0000060 \pm 0,0000020.$$

The circumstance, mentioned above, that the novæ appearing in other nebulae than the Andromeda probably belong to the very brightest of their kind, the estimation of the parallax of this nebula will be regarded as more certain than the mean parallax derived for 7 spiral nebulae, which can only be considered as an upper limit.

If we take into consideration that the Andromeda nebula has a total magnitude as high as that of a star of the magnitude 5,0, it may seem implausible that a nova of the magnitude 7,2 should appear in it (HARTWIG's nova). From CHAPMAN and MELOTTE's star countings we conclude that the stars brighter than the magnitude 21,0 possess an apparent magnitude in total equivalent to that of 690 stars of the first magnitude. Thus our stellar system would, if no changes with respect to the distances of the stars from us were undertaken, and their light were concentrated in one point, shine like a star of the magnitude -6^m . TYCHO BRAHE's Nova 1572 is said to have reached the magnitude -5^m . As we have only to deal with a rough estimate, it ought here to be permitted to suppose that this nova is situated at the same distance as the stars of the galaxy on an average, and thus there is nothing

astonishing in the fact the Nova Andromedæ 1885 reached a magnitude, which differs only by 2 classes of magnitude from the magnitude of the nebula itself.

On the appearance of the Nova Andromedæ 1885 CHARLIER referred to a rare old publication²⁸ which shows that a nova had probably appeared in the Andromeda nebula in 1664. We will give a short account of the contents of this paper.

BULLIALDUS points out how, at the appearance of the comet of 1664, the astronomers discovered a new phenomenon, viz. a nebula which appeared in the belt of Andromeda. According to the author it cannot be counted among the faint and insignificant phenomena, and it has been observed earlier by SIMON MARIUS in 1614, whose observations regarding this object are quoted. With MARIUS the author is astonished that TYCHO BRAHE has not mentioned the Andromeda nebula, and as HIPPARCHOS and HEVELIUS and many others do not seem to have observed it, the author concludes that it appears and disappears alternately. The author has found a manuscript to a star catalogue with drawings of the constellations, and in the representation of Andromeda the unknown drawer, who, according to BULLIALDUS, cannot have been living much earlier or much later than 1500, has represented the nebula with dots in the same manner as he has in the same work drawn the nebula (star cluster) in the Cancer (Præsepe). BULLIALDUS adds:

»From what has been said, I conclude that this nebula has been observed by some astronomers 150 years ago. As, however, it has not been noticed either by HIPPARCHOS or by any other ancient scientists, and neither in the last century has been drawn by TYCHO BRAHE, the generalissimo of the astronomers, nor in our time by BAYER, and as even at this moment, in November 1666, it looks quite obscure and insignificant after two years ago having shone exceedingly bright, it follows with necessity, that it appears and disappears by intervals.»

Any great amount of importance ought perhaps not to be attached to the, nevertheless, curious circumstance that the nebula has not been observed by BRAHE, HEVELIUS, and HIPPARCHOS, but, on the other hand, the author's express statement that the magnitude of the nebula in 1664 was exceptionally high, seems to indicate that just then some nova of extraordinary magnitude had flashed up within the nebula. If we suppose the total magnitude of the nebula to be $5^m.0$, and if we suppose that a nova of the magnitude 6.0 has appeared in the nebula, we conclude that the nebula, during the appearance of the nova, should have had a total magnitude of nearly $4^m.6$. This difference in the magnitude of the nebula should, of course, easily be noticed by an observer accustomed to observe the sky. Besides, nothing prevents us from assuming a still greater upper limit for the maximum magnitude of a nova than was the case with that of TYCHO BRAHE.

9. The Magellanic Clouds.

If we finally start from our already suggested hypothesis that the Magellanic clouds are spiral nebulae (Pl. 1) and the nearest hitherto known, we thereby obtain

a means to derive the distance to the Andromeda spiral, provided the true dimensions are the same in both cases. From variable stars SHAPLEY has found for the small Magellanic cloud $\pi = 0'',000052$, which might be the hitherto most certainly determined parallax-value for this object. As N. G. C. 224 has a diameter of 120' and the diameter of the cloud is about 300' we obtain:

$$\pi_{\text{Andr.}} = 0'',000024.$$

For N. G. C. 371, connected with the small Magellanic cloud, follows, according to the principles of SHAPLEY, $\pi = 0'',000033$, which gives:

$$\pi_{\text{Andr.}} = 0'',000013.$$

Our discussion consequently has given the following result regarding the parallaxes of the spiral nebulae:

N:r	Method	π_m	$\pi_{\text{Andr.}}$	Diameter in light-years for N. G. C. 224
1.	From direct parallax-measures	$\leq 0'',005$	—	> 23
2.	According to JEANS' theory	$\leq 0,0007$	$0'',000080$	$= 1400$
3.	From star counts	$\leq 0,000420$	—	> 270
4.	From WOLF's measurements of the »caves» . .	$\leq 0,000005$	$0,000100$	> 1140
5.	From spectral and photometrical researches . .	small	—	—
6.	From proper motions and radial velocities . .	$\leq \begin{Bmatrix} 0,000284 \\ 0,000163 \end{Bmatrix}$	—	> 510
7.	From the rotational motions	$\leq \begin{Bmatrix} 0,000358 \\ 0,000221 \end{Bmatrix}$	—	> 290
8.	From the novæ in spiral nebulae	$\leq 0,00000024$	$0,000005$	$= 23000$
9.	From the small Magellanic cloud	$\leq 0,0000009$	$0,000018$	$= 6300$

The values in the table calculated for π_m according to the 4th, 8th and 9th methods have been obtained by assuming, in conformity to Tables XIV and XV, that the mean parallax of the known spiral nebulae is $1/21$ of that of the Andromeda nebula.

The first value only indicates that direct parallax-measures cannot be employed for determining the distances of spirals. The third value is an upper limit. To the fourth and following two methods no great importance is here to be attached owing to the uncertainty and the small number of the observations. *The greatest importance must be attached to the parallax-value obtained from the novæ* as no less than 11 novæ have been registered in the Andromeda nebula. I will here add that it is at the calculating of π not necessary to accept the galaxy-theory concerning the Andromeda-nebula. Even if SHAPLEY'S²⁰¹ view is adopted that the novæ in spirals may be considered as the engulfing of a star by rapidly moving nebulosity, the appearance of novæ in spirals might nevertheless be employed for an estimation of the mean

parallax. The only doubtful hypothesis to be introduced is that the mean maximum absolute magnitude is the same for the novæ in the Milky Way as well as for those in the spiral nebulæ, and at present there is nothing to be found that speaks against such a supposition.

The last parallax-value is of interest as being of the same order of magnitude as the preceding one. Considering the somewhat uncertain suppositions on which its calculation is based I have nevertheless not wanted to assign to it any great importance.

The present investigation has given as the main result that the spiral nebulæ must be considered as situated at considerable distances from the solar system. Whether they are Jeans' star-producing mechanisms or remote galaxies is, on the other hand, more difficult to decide. Possibly we might in the present facts see a suggestion that the latter is the case, but the spiral nebulæ do not, however, seem to be of such dimensions as those that should be ascribed to the galactic system with regard to Shapley's investiga-

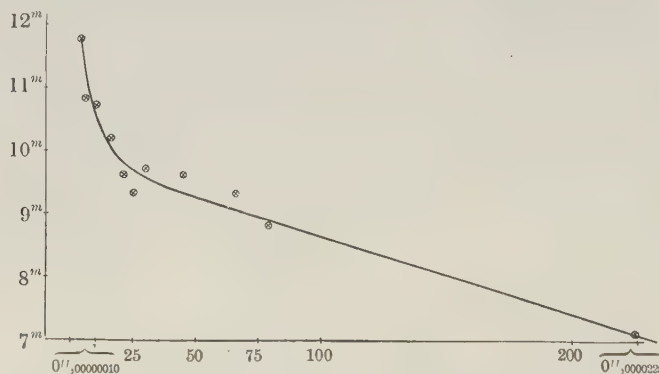


Fig. 11. The relation between parallax and total magnitude for spiral nebulae.

tions, and much also speaks against regarding the Galaxy as having a structure, analogous to that of the spiral nebulae.

Finally I have, under the supposition that the true dimensions of spiral nebulae are on an average of the same size, derived relative parallaxes for the large spirals from measurements of their apparent diameters. Certainly this supposition is less justified here than concerning globular clusters, as it seems to be evident, that spiral nebulae may be divided up into several types, but on an average the supposition will nevertheless be correct. Another mean of obtaining the relative parallax we have in the estimations of the total magnitude of nebulae, performed by HOLETSCHEK, KRITZINGER, STONE, and the STRASSBURG OBSERVERS. The total magnitude of a nebula is a function of its surface intensity, its extension and its condensation, and the smaller the nebula is, the more the total magnitude tends to approach the conception of star magnitude. Therefore it is evident that the total magnitude does not vary with the parallax in the same way as the apparent diameter. The relation — given in the curve above — between π , calculated from the apparent diameters, and the total magnitude m , is derived from the 59 nebulae in the table XIV.

The tables XIV and XV, without aiming at completeness, contain the nebulae that are known as spirals and for which observational data of the diameter d and m are available.

With the adopting of

$$\pi_{\text{Andr.}} = 0'',0000050$$

the absolute parallaxes have been derived from the diameter-values according to the formula:

$$\pi = 0'',000000417 \ d$$

and from m according to the curve in Fig. 11.

The values for m have been corrected for systematic errors in accordance with WIRTZ' discussion and reduced to HOLETSCHEK's scale.

The values for the diameters are taken from the surveys of PEASE, FATH, CURTIS, KNOX SHAW, BAILEY, WOLF and KRITZINGER. The values given by BAILEY are corrected for systematic errors because his values are systematically too small. The present writer has further measured the diameter from published photographs and in a few cases from plates of his own.

It stands to reason that the values for π_m might be considerably changed by future investigations, when more detailed observations will have been made regarding the nebulae. I have nevertheless given the parallax-values in the tables, XIV and XV in the hope that they might have some value as a first attempt of reconnoitring the wonderful system of spirals which obviously seems to be connected with the globular clusters and with the Milky Way, a connection, the nature of which is not yet satisfactorily known.

Upsala, Astronomical Observatory, 15th May, 1919.

Table XIV.

Magnitudes, Apparent diameters and Hypothetical parallaxes for 59 Spirals.

N. G. C.	m					d										Parallax	
	Holet- schek	Krit- zinger	Strass- burg	Stone	Mean	Pease	Fath	Curtis	Shaw	Bailey	Wolf	Krit- zinger	Lund- mark	Mean		From m	From d
205	9 ^m ,2				9 ^m ,2	10'			8'					9'		0'',00000050	0'',00000037
253	8,8	7 ^m ,5			8,3					20'				20		105	83
278	10,7				10,7	1,4								1,4		8	6
470	11,8				11,8			1,6						1,6		3	7
598	7,0	7,2			7,1		58,7			—	54'		50'	54		225	225
613	9,6				9,6			4	5				5	5		29	21
628	9,6	10,2			9,9								7	7		14	29
908	10,6				10,6			3,5					4	3,7		9	15
936	10,0	9,9		—	10,0			1,5	3					1,8		16	7
955	11,6			11,0	11,3	1,3								1,3		4	5
1068	8,7			7,6	8,1	6							2?	4		125	17
1084	10,2	10,4			10,3			2,5						2,5		12	10
1637	11,3		11,7		11,5			3						3		5	12
2403	8,8	—			8,8	20				—				20		88	83
2683	9,2	9,4	—		9,3								5	5		45	21
2841	9,1				9,1	6,5							6	6,2		60	26
2903	9,0	9,0			9,0					—	8		4	6		70	25
2976	10,7	10,2			10,5	3,2		3,2					3,5	3,2		9	13
3031	8,0	7,7			7,9					11	18		20	18		138	75
3034	8,8	9,5			9,2					11			7	9		50	38
3079	10,7		9,8		10,3			8					6	7		12	29
3169	11,0				11,0			4						4		7	17
3184	11,4	9,7			10,5			6					6	6		10	25
3198	11,7				11,7								6,6	7		4	29
3226	10,7		12,1		11,4								3	3		5	12
3556	10,0	—			10,0								7	7		16	29
3623	8,9	—			8,9								7,2	7		75	29
3627	8,6	—			8,6								6,4	6		101	25
3628	10,2		9,8		10,0								12	12		16	50
3666	11,5				11,5	3,5								3,5		5	15
3681	11,7				11,7			1						1		4	4
3684	11,7				11,7			1,2						1,2		4	5
3686	10,7				10,7			2						2		20	8
3729	10,5				10,5			1,5						1,5		10	7
4088	10,3		9,8		10,0								5	5		16	21
4192	9,8				9,8			14						14		20	58
4244	11,0				11,0								14	14		7	58
4258	8,7	—			8,7								8	8		90	33
4321	9,8	—			9,8					5			5	5		20	21
4449	9,0		8,7		8,8	4,5				6,5			4	5		88	21

N. G. C.	<i>m</i>					<i>d</i>										Parallax	
	Holet- schek	Krit- zinger	Strass- burg	Stone	Mean	Pease	Fath	Curtis	Shaw	Bailey	Wolf	Krit- zinger	Lund- mark	Mean	From <i>m</i>	From <i>d</i>	
4485	10,8		10,9		10,8								5	5	0,00000006	0,00000021	
4565	9,4				9,4						15		15	15	35	62	
4594	8,7		8,7	—	8,7	7				9			6	7	118	29	
4656	10,5				10,5			12		—			10	11	9	46	
4736	7,7				7,7	5				6			7	6	178	25	
4900	10,5				10,5	1,5		1,2					1,9	1,5	9	7	
5005	9,1	8,7			8,9	3,5				—		3		3,2	75	13	
5194	7,7	7,7			7,7			1			10		9	10	178	42	
5236	9,5			—	9,5				10	12				11	31	46	
5247	12		12,2		12,1			5	4					4,5	2	19	
5457	9,2	8	7,8		8,5						18		16	17	108	71	
5713	11,0				11,0		0,9							0,9	7	4	
5866	10,3		9,0		9,6	2,5				2,0				2,2	27	9	
5907	10,6		9,8		10,2	11		11						11	14	46	
6217	10,8		9,8		10,3	1,5								1,5	12	7	
6946		10,4			10,4						7	20	12	13	11	54	
7457	11,5		10,9		11,2			2						2	6	8	
7537			10,9		10,9			2						2	7	8	
7782			12,1		12,1								1	1	0,00000003	0,00000004	

Table XV.

Magnitudes, Apparent diameters and Hypothetical parallaxes for 216 Spirals.

N. G. C.	Magnitude					Diameter								Parallax Unit is 0'',00000001		
	Holet-schek	Krit-zinger	Strass-burg	Stone	Mean	Pease	Fath	Curtis	Shaw	Bailey	Lund-mark	Hard-castle	Mean	From <i>m</i>	From <i>d</i>	Mean
29			12 ^m ,8		12 ^m ,8			1',2					1',2	2	5	3
55									25'	25'			25		104	104
68			13,0		13,0			1,5					1,5	1	6	4
83			12,0		12,0			1					1	2	4	3
134									5				5		21	21
151								4					4		17	17
157	10 ^m ,2		10,9		10,5			2,5					2,5	9	10	9
246									4		4',2		4,1		18	18
247								16	18				17		71	71
289									2				2		8	8
300									14			20'	17		71	71
488	10,5	9 ^m ,1	—		9,8			3			2		2,5	20	10	15
492								1					1		4	4
514								2					2		8	8
650,1	9,4				9,4	3',0					1,5		2,2	39	10	25

N. G. C.	Magnitude					Diameter									Parallax Unit is 0'',00000001		
	Holet- schenk	Krit- zinger	Strass- burg	Stone	Mean	Pease	Fath	Curtis	Shaw	Bailey	Lund- mark	Hard- castle	Mean	From <i>m</i>	From <i>d</i>	Mean	
676	<12,0				12									2		2	
693			12,0		12									2		2	
740								1,1					1,1		5	5	
761								1					1		4	4	
765								1					1		4	4	
772	10,0				10			5					5	16	21	18	
891											10		10		42	42	
895								4					4		17	17	
941								2	—				2		8	8	
972	12		12,0		12			—				8	8	2	33	17	
1023	9,7				9,7	5		6					6	23	25	24	
1097									8				8		33	33	
1186			12,0		12,0									2		2	
1187				—					2				2		8	8	
1232				11,4	11,4							6	6	4	25	15	
1300								6					6		25	25	
1337								6					6		25	25	
1365									7				7		29	29	
1530			12,0		12		2,5						2,5	2	10	6	
1532									5				5		21	21	
1599								0,7					0,7		3	3	
1642								1,7					1,7		7	7	
1659			10,9	12,0	11,5									4		4	
1667			10,9		10,9									7		7	
1700	11,2			—	11,2									6		6	
1784								1,5					1,5		6	6	
1792									3				3		12	12	
1808									4				4		17	17	
1964									4				4		17	17	
2207			—	—					3				3		12	12	
2371,2	{10,7,}		—		10,1	4		2					3	15	12	13	
2442	{11,0}											6	6		25	25	
2655	10		—		10			4					4	16	17	16	
2681	9,7				9,7	1,2		0,6					0,9	23	3	13	
2768	9,7				9,7			1,4					1,4	23	6	15	
2825			12,5		12,5									2		2	
2830	10,5		13,0		11,7									4		4	
2835									6				6		25	25	
2968	11,3				11,3									6		6	
2997									6				6		25	25	
2998			12,2		12,2		0,9						0,9	3	4	3	
3003	12		10,9		11,5			5					5	4	21	13	
3109									10				10		42	42	
3115	9,0	9,1		—	9,0									71		71	
3166	11,7				10,7			5					5	8	21	14	

N. G. C.	Magnitude					Diameter								Parallax Unit is 0'',00000001		
	Holet- schek	Krit- zinger	Strass- burg	Stone	Mean	Pease	Fath	Curtis	Shaw	Bailey	Lund- mark	Hard- castle	Mean	From <i>m</i>	From <i>d</i>	Mean
3254	11,5				11,5		4,5						4,5	4	19	12
3346	11,0				11,0			2,5					2,5	7	10	8
3368	8,7				8,7			7					7	95	29	62
3389	11,8				11,8			2					2	3	8	5
3395	11,3	11,4	—		11,3									6		6
3423			13,0		13,0									1		1
3511									4				4		17	17
3513									2				2		8	8
3521	9,3	—	—		9,3					4			4	46	17	32
3593	10,6		—		10,6	4,5								4		4
3596	<12		10,9		11,5									5		5
3621									5				5		21	21
3627	8,6		—		8,6						6,4		6,4	103	27	65
3631	10,5				10,5						3,6		3,6	9	15	12
3672	11,7		10,9	12,4	11,7				3				3	4	12	8
3675	10,2				10,2									14		14
3681	11,7				11,7			1					1	4	4	4
3684	11,7				11,7			1,2					1,2	4	5	4
3686	10,7				10,7			2					2	8	8	8
3691			12,5		12,5			1					1	2	4	3
3705			10,9		10,9									7		7
3718								3					3		12	12
3726											5,6		5,6		23	23
3729	10,5				10,5			1,5					1,5	9	6	7
3887	11,0		10,9	—	10,9				2				2	7	8	7
3893	10,5				10,5									9		9
3935			11,4		11,4									4		4
3938	10,8		—		10,8	4							4	8	17	12
3953	10,0		—		10,0									16		16
3971			11,6		11,6									4		4
3986			12,0		12,0									2		2
3991			12,2		12,2									2		2
3994			11,4		11,4									4		4
3995			11,6		11,6									4		4
4013	12				12									2		2
4016										1,2			1,2		5	5
4020	11,0				11,0									7		7
4030	10,0				10,0									16		16
4051	10,0				10,0									16		16
4062	11,3				11,3									6		6
4150	10,7		—		10,7									8		8
4186			13,0		13,0									1		1
4206						4							4		17	17
4208							0,2						0,2		1	1
4212	11,0				11,0		2,0						2,0	7	8	7

N. G. C.	Magnitude					Diameter									Parallax Unit is 0'',00000001		
	Holet- schek	Krit- zinger	Strass- burg	Stone	Mean	Pease	Fath	Curtis	Shaw	Bailey	Lund- mark	Hard- castle	Mean	From <i>m</i>	From <i>d</i>	Mean	
4214	9,3		9,8		9,5									33		33	
4216	9,6			9,6	9,6	6							6	28	25	26	
4220	10,8		—	10,8	10,8									8		8	
4236	11,5				11,5									5		5	
4254	9,5		9,2		9,3		4,3			5	4,3		4,5	46	19	33	
4274	10,0		—		10,0									16		16	
4303	9,4	9,0	—		9,2					7	5		6	53	25	39	
4304									1				1		4	4	
4346	10,7				10,7									8		8	
4361	9,4			—	9,4									39		39	
4374	8,7				8,7									95		95	
4382	8,8		—		8,8			4		6			5	86	21	54	
4388						3,5							3,5		15	15	
4394	10,3	8,2			9,2			4					4	53	17	35	
4402						3							3		12	12	
4406	9,0				9,0	1,2				—			1,2	69	5	37	
4414	9,3		—		9,3									46		46	
4425			10,9		10,9									7		7	
4429	10,3		—		10,3									12		12	
4438	9,8				9,8									21		21	
4448	10,5		9,8		10,1									15		15	
4450	9,6		9,8		9,7					5			5	23	21	22	
4472	8,5	—	9,8		9,2					6			6	53	25	39	
4486	9,0		8,7		8,8					6			6	86	25	56	
4501	9,5		9,8		9,6					3			3	28	12	20	
4517	11,2				11,2									6		6	
4526	10,0	—			10,0					2			2	16	8	12	
4536	11		9,8		10,4						7		7	11	29	20	
4548	10,0		—		10,0									16		16	
4559	9,7				9,7		0,6				8		4,3	23	18	20	
4567						2							2		8	8	
4568						4							4		17	17	
4579	8,8				8,8									86		86	
4618	11,0		10,9		10,9									7		7	
4631	9,1		—		9,1					12	15		13,5	62	56	59	
4632	11,8		12,0		11,9		2,3						2,3	3	10	6	
4639			10,9		10,9									7		7	
4642							1,4						1,4		5	5	
4651			9,8		9,8									21		21	
4666	10,7		—		10,7		4,5						4,5	8	19	13	
4710	10,5		9,8		10,1						8		8	15		15	
4725	8,7				8,7									95	33	64	
4762	10,3		9,8		10,0									16		16	
4750									1				1		4	4	
4826	8,6				8,6					8	8		8	103	33	68	

N. G. C.	Magnitude					Diameter								Parallax Unit is 0'',00000001		
	Holet- schek	Krit- zinger	Strass- burg	Stone	Mean	Pease	Fath	Curtis	Shaw	Bailey	Lund- mark	Hard- castle	Mean	From m	From d	Mean
4966							0,7						0,7		3	3
4981								1,5					1,5		6	6
5000							1,7						1,7		7	7
5033	9,6	8,4			9,0									69		69
5055	9,2	7,4			8,3					6	8		7	128	29	79
5128										10			10		42	42
5183										1,2			1,2		5	5
5184										1,0			1,0		4	4
5211			12,2		12,2					0,7			0,7	3	3	3
5236	9,5			—	9,5				10		10		10	33	42	38
5247	12		12,2		12,1			5	4				4,5	3	19	11
5248	10,3				10,3									11		11
5254			12,5		12,5									2		2
5258										1,2			1,2		5	5
5308	11,0				11,0									7		7
5383	11,3				11,3									6		6
5394	<12				12									2		2
5426			12,5	11,9	12,2				1				1	3	4	3
5427			12,0	11,6	11,8				2				2	3		3
5483															8	8
5544						1,2							1,2		5	5
5545						0,8							0,8		3	3
5691										0,8			0,8		3	3
5716		12,5			12,5				—					2		2
5728		10,7		10,6	10,7				—					8		8
5740	11,0		12,0		11,5					1,5			1,5	4	6	5
5746	9,5		9,8		9,7					6			6	23	25	24
5850								2,2		1,5			1,9		8	8
5921	11,5	11,2			11,3			5					5	6	21	13
5956								0,6					0,6		2	2
5957								1					1		4	4
6181	11,2		—		11,2			2					2	6	8	7
6217	10,8		9,8		10,3									12		12
6381							0,9						0,9		4	4
6412						1,5		2					1,8		7	7
6478			10,9		10,9									7		7
6555			—					2					2		7	7
6574			10,9		10,9			1,4					1,4	7	5	6
6643	10,6	10,7			10,6			3					3	9	12	10
6814			10,9	9,8	10,3			2					2	12	8	10
6951			—					3,5					3,5		15	15
7129								5					5		21	21
7156								1					1		4	4
7177						2,5		3					2,8		12	12
7184								5					5		21	21

N. G. C.	Magnitude					Diameter								Parallax Unit is 0'',00000001		
	Holet- schek	Krit- zinger	Strass- burg	Stone	Mean	Pease	Fath	Curtis	Shaw	Bailey	Lund- mark	Hard- castle	Mean	From <i>m</i>	From <i>d</i>	Mean
7217						3							3		12	12
7293								13	15				14		58	58
7314									2,5				2,5		10	10
7331	9,3	—			9,3					—	2,8		2,8	46	12	29
7371								1					1		4	4
7418									2				2		8	8
7421									1,5				1,5		6	6
7448	10,5				10,5			1,6					1,6	9	7	8
7479	10,8				10,8						4		4	8	17	12
7497								6					6		25	25
7541	11,4		10,9		11,2			3					3	6	12	9
7640								9					9		38	38
7721			12,2	11,4	11,8				2,5				2,5	3	10	6
7752								2,5					2,5		10	10
7755									1				1		4	4
7769			11,4		11,4									4		4
7771			9,8		9,8									21		21
7782			12,1		12,1									3		3
7793									6				6		25	25
7814	10,3		9,8		10,1						1?		1	15	4	9
7817			10,9		10,9									7		7

In Table XVI we have grouped the 276 nebulae of Tables XIV and XV according to their parallaxes and calculated the numbers of nebulae per volume-unit ($=10^{18}$ parsec³).

Table XVI.

π	Number of Nebulae	Density
0'',00000401—0'',00000500	1	15,4
301 400	—	—
201 300	1	3,9
101 200	4	1,5
91 100	2	1,4
81 90	3	1,4
71 80	6	1,5
61 70	7	1,3
51 60	6	1,0
41 50	10	0,7
31 40	18	0,4
21 30	28	0,2
11 20	62	0,05
6 10	65	0,01
0'',00000001—0'',00000005	63	0,0001

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Abbreviations.

A. N. Astronomische Nachrichten.
 A. J. Astronomical Journal.
 Aph. J. Astrophysical Journal.

M. N. Monthly Notices.
 H. A. Harvard Annals.
 P. A. Popular Astronomy.

Appendix.

During the time that has passed since the present investigation was elaborated, I have continued my studies on the same subject. The results will be published this year as an »Ergänzungsheft» to the *Astronomische Nachrichten*.

In a preliminary notice we will here communicate some of the more important results.

§ 1.

The relative parallaxes of the globular clusters have been determined also with aid of other data than the apparent diameter as an equivalent of distance. Thus I have for this purpose compiled, and, where possible, collected by observations of my own, new data regarding the star number, total magnitude, maximal magnitude, visual and photographic diameter &c., of the globular clusters. The diameters of the greater clusters it has been possible to define strictly with aid of the curves of VON ZEIPPEL²⁶⁰, giving the relation between the distance from the center of the cluster and the star density, determined from photographic plates.

In this respect I have made extensive use of a copy of the Harvard Sky, belonging to the Lund Observatory. I beg here to express my thanks to Professor CHARLIER for the benevolence with which he has placed the plates at my disposal.

The new parallax-values generally confirm the values of SHAPLEY and those derived above by the present writer. The absolute parallax-values found by us also have gained confirmation, especially by a comparison between SHAPLEY's values for m_{25} and the absolute magnitude of the red giants of our stellar system, derived as well from direct parallax-measurements as from ADAMS' and JOY's spectroscopical parallaxes.

§ 2.

With the publication in SHAPLEY's 14th paper of radial velocities of spirals, determined by SLIPHER during the last years, there are presently at hand such values for 29 spirals. A determination of the Apex of solar motion by means of the method used in Section II have given the following results:

I	II
$A = -15^\circ$	$A = -44^\circ$
$D = +61^\circ$	$D = -18^\circ$
$V_0 = -517 \text{ km./sec.}$	$V_0 = -813 \text{ km./sec.}$
$K = +587 \text{ km.}$	$K = 0.$

Regarding the interpretation of the constant K we want to call attention to the circumstance that it corresponds to a displacement of the spectral lines in the same direction as that demanded by the general theory of relativity of EINSTEIN. This displacement is of the amount

$$0,634 M^{2/3} \rho^{1/3} \text{ km./sec.},$$

where M designs the mass of the celestial body emitting the spectre and ρ its density.

It might be uncertain if it is justified to use this formula with regard to the spirals, as their spectrum ought probably to be considered as a resultant of individual spectra of a great number of stars. If we accept nevertheless the validity of the formula, we obtain, putting $\rho = 10^{-19}$, and if $K = 587 \text{ km.}$ is assumed as corresponding exclusively to an Einsteinian gravitation-effect:

$$M = 0,9 \cdot 10^{13} \odot.$$

This result shows that the observed displacement exceeds by at least 1000 times the value one might expect by assuming a value of M agreeing with our present knowledge of the spirals. The Einsteinian theory however permits an explanation of this displacement by taking into consideration the total field of gravitation acting upon the light throughout its way to the Earth.

§ 3.

Calculations of the proper motions of a number of the larger spirals, based on all existing meridian observations and micrometrical measures, give somewhat smaller values of the mean proper motion than those used in Section III. Probably the proper motions derived from visual measurements as well as those obtained from photographic determinations are illusory and indicate only the accuracy of measurement of the method employed. Thus CURTIS finds in his investigation mentioned above⁴⁶:

Class of objects	Mean proper motion
Large diffuse nebulae	$0'',036$
Planetary and gaseous nebulae	$0,028$
Very small nebulae (spirals?).	$0,040$
Spirals	$0,033.$

Nor do the proper motions of small nebulae (spirals?) calculated by WIRTZ seem to depend on the magnitudes or diameters of the spirals, which indicates that

they must also be regarded as illusory. Nevertheless the preceding calculations of mean parallaxes of spiral nebulae, derived from their proper motions and radial velocities, will have their value as fixing an upper limit of the parallaxes.

Since the above paper was written, the discovery of four new stars in the Andromeda nebula has been communicated. If the $M_{\max.}$ of the 15 novæ known as yet in this nebula is used for a determination of its parallax, we find a value only slightly differing from that previously obtained. There seem to exist several reasons for assuming that the novæ also regarding the $M_{\min.}$ belong to both the giant and dwarf types. In the Andromeda nebula only HARTWIG's Nova would be of the giant type and the 14 others dwarfs, whereas in the other nebulae it seems plausible that it is only giant novæ that we have been able to observe. With this assumption, we obtain the parallax-values given under the head » π from novæ» in the table below.

At determining the apparent star density on plates of the Andromeda nebula, a greater average density seems to occur within the nebula than outside, provided that sufficiently faint stars are included in the counts. This peculiar instance probably ought not to be interpreted in any other way than as indicating an incipient dissolution of the Andromeda system into separate stars. As all stars belonging to this system are at the same distance from us, it is evident that we can see only those stars dissolved that have the greatest luminosity. If we assume that the absolute magnitude attains the same maximum in the Andromeda system and in the other spirals as in our stellar system, we have herein, if this maximum can be determined, a new means of finding the parallax of those spirals for which we have been able to observe an incipient dissolution into separate stars.

The direct parallax-measurements of stars indicate that the absolute magnitude can attain an upper limit of -7^m . Departing from the parallax determined for the small Magellanic cloud from δ Cephei-variables, cluster-diameters and KAPTEYN's integral-equation (3), and calculating the absolute magnitude of its most brilliant stars, we find that they attain a maximal magnitude of -6^m . The data given by RUSSELL¹⁷¹ regarding the frequency of absolute star magnitudes also give an upper limit agreeing with this value.

This quantity evidently can not at present be determined with any greater accuracy, but we will provisionally assume the value -6^m .

On plates taken with the 6-inch twin telescope of this Observatory with exposures between 6 sec. and $9\frac{3}{4}$ hours, star counts have been made, showing at a provisional reduction that the dissolution of the Andromeda nebula into separate stars is beginning at the star-magnitude 14. We thus obtain:

$$\pi_{\text{Andr.}} = 0'',000010,$$

a parallax only two times as great as that found from the novæ.

Professor VON ZEIPPEL kindly has put at my disposal a plate of Messier 33 taken on Mount Wilson at the 60-inch telescope with an exposure of 8,5 hours. On

this plate the outer parts of the nebula appear distinctly dissolved, and although some nebulous stars make it difficult to distinguish the stars that belong to the nebula, it is indubitable that we can here distinguish several thousands of the stars that form a part of the vast stellar system represented by Messier 33.

In the following Table XVII are given the parallaxes obtained by different methods for some of the larger spiral nebulae.

Table XVII.

N. G. C.	π			
	From novæ	From star counts	From apparent diameter	From magnitude
221	0'',0000050	0'',0000100	(0'',0000050)	(0'',0000050)
598	—	0,0000032	0,0000023	0,0000023
3031	—	0,0000016	0,0000008	0,0000014
6946	0,00000019	0,00000080	0,00000054	0,00000011
2841	0,00000010	—	0,00000026	0,00000060
4321	0,00000029	—	0,00000021	0,00000020

As Mr. B. LINDBLAD has kindly pointed out to the present writer, the spectrum of the Andromeda nebula, with regard to the intensity of the iron bands $\lambda 382^{\mu\mu}$ and $\lambda 388^{\mu\mu}$, shows such great analogies with the spectra of giant stars that we may be entitled to assume that the spectrum is generated by giants.

If it were possible to determine the apparent magnitude at which the main body of giants in a spiral nebula begin to appear separated, it would give us an additional means of estimating its distance.

It seems plausible that regarding the Andromeda nebula this separation ought to appear at the 20th or 21st magnitude, and if we assume the absolute magnitude of the giant series to be, in the average, $=0^m$, we obtain a value of the parallax agreeing comparatively well with the other determinations.

Photographs on a large scale of N. G. C. 224 and 598, that have been placed at the disposal of VON ZEIPPEL by Mr. ADAMS, show a great number of rifts (caves) in the spirals. These rifts, with regard to their external structure, are very similar to those formations in the Milky Way that have been investigated above all by the extensive researches of BARNARD.

A catalogue, now including 560 objects, has been made of all nebulae that are undoubtedly belonging to the class of spirals. Their distribution in the sky and in space has been investigated, besides which some other statistical researches have been made concerning such observational data as have been available regarding these objects.

§ 4.

In order to give a contribution to the question of the structure of the Milky Way, I have during the later years undertaken an investigation of the star-distribu-

tion in the great star-cloud in Cygnus. On plates taken with the Upsala 6-inch twin telescope the number of stars within each magnitude has been determined for stars between the 9th and 15th magnitude. Nearly 500 000 stars are thus classified and provisional results have been derived. A preliminary determination of the parallax of the cloud by using the results of the star-counts and the formula (3) gives the following value:

$$\pi = 0'',00015.$$

PANNEKOEK in M. N. 79: 500, 1919 has found a considerably smaller value, viz.

$$\pi = 0'',000025.$$

Most classes of objects that must be considered as related to the galactic clouds cannot be regarded as being at greater distances than 1000—10000 light-years. This especially is the case with the novæ and O-stars.

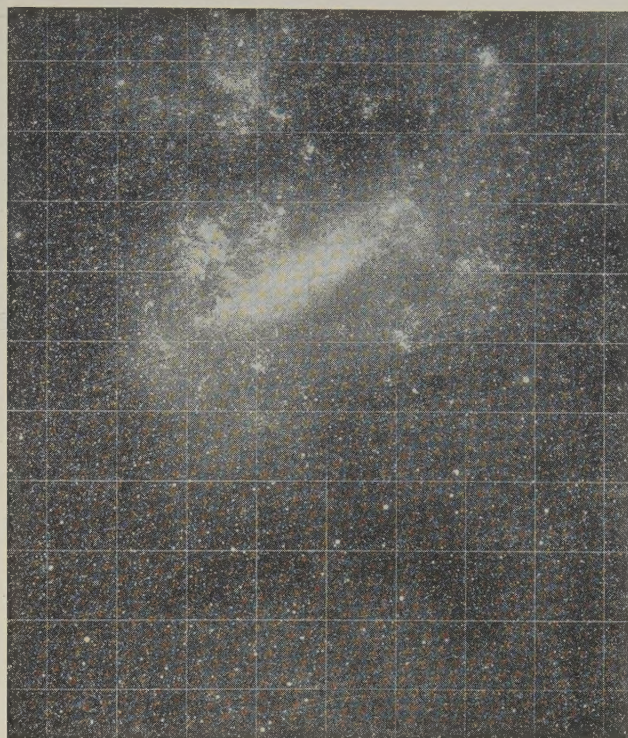
By use of the same methods as those employed for the globular clusters, I have studied the distribution in space of the open clusters, which evidently form a skeleton of the Milky Way. The distances thus found give an average of 15 000 light-years.

Several other facts are also indicating that the distance to the galactic clouds cannot be regarded to be as great as is demanded by assuming the globular clusters as being related to the Milky Way. To be sure, a considerable number of globular clusters appear to be intimately connected with the Sagittarius-cloud, but nevertheless it seems not at all plausible that the extent of the Milky Way should be as great in other directions, and it appears questionable whether already 100 000 light-years is not too great a value for its diameter.

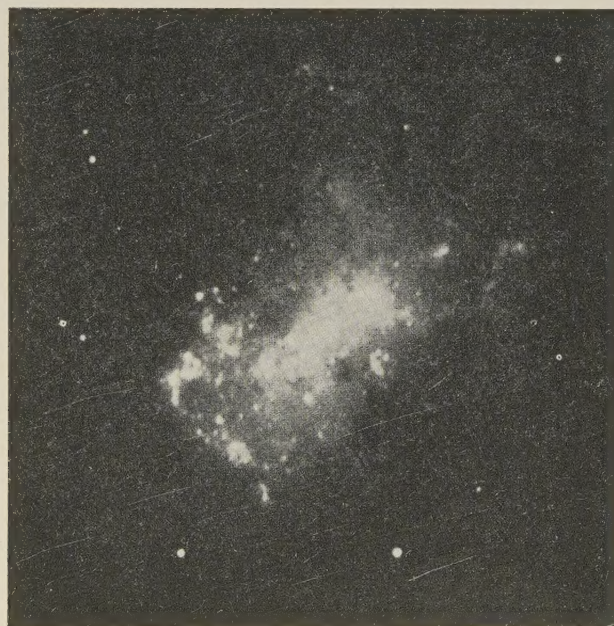
Upsala, Astronomical Observatory, Jan. 26th, 1920.



Tryckt den 5 februari 1920.



Photograph of the Large Magellanic Cloud, reproduced from *Stellar Photographs* by the late Mr. Franklin-Adams.



The spiral nebula N. G. C. 4449, reproduced from *Pease, Photographs of nebulae 1911—16.*

